One-Shot Coordination of Feeder Vehicles for Multi-Modal Transportation

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***Abstract*— In this paper, we consider coordinated control of feeder vehicles in the first leg of a multi-modal transport system. In particular, we consider a one-shot problem wherein the passengers must be transported to a common destination before a fixed deadline. We pose the problem as the maximization of the service provider's profits through optimal pricing and feeder *allocations* given the knowledge of demand and supply distributions. Though the original problem is non-linear, with optimal pricing scheme we reduce it to a linear program. We then move on to developing an off-line route elimination algorithm that reduces the problem size in terms of computation memory and time. Further, we provide a simplified way to compute the maximum possible profits as a function of total supply for a given demand distribution. We test the framework with simulations on a 20 node graph under a fixed-demand profile.**

I. Introduction

Real-time, demand and supply aware, coordination of a fleet of vehicles has the potential to improve the efficiency of multi-modal transportation systems, such as public transportation, freight transportation networks and supply chains. For example, a coordinated fleet could effectively address the challenges faced by a traditional public transportation such as sparse coverage, lack of first or last-mile connectivity, lack of convenience and longer travel times. In this paper, we address the problem of "first-mile" coordination of feeder vehicles of a multi-modal transportation system. In particular, we focus on *one-shot* coordination where a one-time single destination demand arises and needs to be transported in a fixed time window. Multi-modal freight transportation networks [1], peak-hour single destination para-transit [2], [3], overnight delivery chain systems [4] and special event management with large foot-fall rely heavily on large-scale pick-ups and single destination drop-off within a fixed time window.

Literature review: The vehicle routing problem [5]–[8] considers pick up and delivery of entities using one or more vehicles whose routes start and end at a depot. It is in general a mixed-integer problem and can be thought of as an extension of the Travelling Salesman Problem. Ride sharing [9]–[11] and dial-a-ride problems [12]–[14] try to match itineraries and thereby design a shared taxi pooling service in a deterministic or stochastic environment given a known or stochastic demand. A special case is the peak-hour single-destination para-transit service, which [3] models heuristically. Taxi-dispatch models [15], [16] match demand and supply with a spatio-temporal metric and optimally allocate taxis based on geographical location of taxis and customer requests. [17] studies economical aspects of a multi-hop ride sharing scheme for peer-to-peer travel. Another similar class of problems is that of demand anticipative mobility [18]–[22], in which routing and rebalancing vehicle flows seek to match demand and supply. These papers consider fixed demand rates at each station for other stations and then design steady state load-balancing and routing flow rates that seek to maximize passenger throughput without increasing network congestion.

Contributions: In this paper we consider the problem of designing a *one-shot* coordinated feeding for the first leg of a multi-modal transport system. We pose the problem in a macroscopic setting by modeling a region of interest as a graph and modeling vehicles and passengers through flows of vehicles, and volumes of demand and supply. The aim for the feeder vehicles is to drop-off passengers at a node designated as the *destination* in the graph, before a fixed time, from where the onward journey can commence. We pose a non-linear problem for maximizing the earned profits that can be reduced to a linear program by modelling prices based on the value of time [23], [24]. Also we show that given the network, the properties of routes can be exploited to reduce the number of routes and thereby the number of optimization variables itself. With further analysis, we give a simplified method for computing the maximum possible profits as a function of total supply for a fixed demand distribution. As we are interested in a one-shot or single-event problem, we pose it as a fixed horizon control problem. Such a setup is applicable to scenarios where the separation between the events is much longer compared to the first leg transit times.

Our problem differs from vehicle routing problems as it doesn't utilize a depot and origin of a feeder flow could be any node where supply is available. Ride sharing and taxi dispatch do not address fixed horizon planning necessary for the one-shot scheme we propose. Demand anticipative mobility problem in [18]–[22] is concerned with designing steady state flows. The taxi para-transit in [3] though similar in nature has a heuristic approach with numerical simulations and formulations with no fixed time window.

Notation: We let $\mathbb{Z}$ denote the set of integers. We use the notation $[a,b]_{\mathbb{Z}}$ and $(a,b)_{\mathbb{Z}}$ to denote $[a,b]\cap\mathbb{Z}$ and $(a,b)\cap\mathbb{Z}$, respectively. We use similar notation for half-open/half-closed intervals.

Note on proofs: Due to space limitations, we have skipped the proofs of the results. These proofs would appear in a more comprehensive journal version.

II. Problem Setup

In this section, we setup the various elements of the coordinated feeder service problem. We start with a graph

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model of the area that we want to design the feeder service for. Then, we describe the decision variables and constraints and conclude the section by posing the profit maximization problem from the service provider's point of view.

A. Graph Model

Let us assume that we are given an abstract macroscopic graph model $G := (V, E)$ of an area for which we want to design the feeder service. Let D be the common destination of the feeders. Each node $l \in V$ has a certain volume of demand for D, d_l, and a volume of supply of feeding vehicles S_l. We assume the demand at D is zero, that is $d_D = 0$. To each edge $(l, k) \in E$, we associate edge weights (t_{lk}, ρ_{lk}), which represent the average commute time and cost of traversal per unit flow on that edge and are strictly positive. Let T be the time on or before which the feeders, starting at time 0, need to transport the passengers to D, which makes this is a one-shot problem.

Remark II.1. *(Graph parameters and knowledge of them). We assume that T is small enough for there to be no substantial change in background traffic conditions. As a result, we assume that the various parameters associated with the nodes and edges of the graph are fixed and known to the designer. Also, it may seem fair to assume that the supply is fully matched to demand. However, an accurate demand distribution may not be available in time and when the actual demand is revealed, the supply may not match it.*

We define a route r as a *walk* in G. Given a route r, V_r is the sequence of nodes along the route, with $V_r(i)$ being the i^{th} node on the route r. Thus,

$$V_r : [1, n_r]_{\mathbb{Z}} \to V, \quad (V_r(j), V_r(j + 1)) \in E, \tag{1}$$

where n_r is the number of nodes (possibly repeating) on the route r. We denote the *origin of the route* r by $o_r := V_r(1)$. The destination of the route r is $V_r(n_r)$. In particular, we are interested in those routes, whose destination is D, that is, $V_r(n_r) = D$. Similarly, E_r is the sequence of the edges that the route r takes to reach its destination, that is,

$$E_r : [1, n_r - 1]_{\mathbb{Z}} \to E, \quad E_r(j) = (V_r(j), V_r(j + 1)). \tag{2}$$

We define the *set of feasible routes* $\mathbf{R}$ as the set of all routes that terminate at the common destination node D and on which the total travel time is less than T. Thus,

$$\mathbf{R} := \{r \mid (1) - (2), \sum_{(l,k)\in E_r} t_{lk} \le T, \ V_r(n_r) = D\}. \tag{3}$$

If no route $r \in \mathbf{R}$ is such that $o_r = l$, then in fact there is no route $r \in R$ that passes through l. Thus, we can exclude such nodes from the graph G without any consequence on the optimization problem we pose in the sequel.

A route can make multiple visits to D. Therefore, we define a *leg* as the journey between two consecutive drop-off's at D or a journey from o_r to D. Hence, D can occur at-most twice in a leg. The leg i of r is denoted as $r^i = (V_r^i, E_r^i)$ in a similar fashion as for r. For a route r, we call leg 1 as its *primary leg* and its remaining legs as *secondary legs*. We denote θ_r as the *number of legs in a route* r. For a route r, we define *per unit flow cost of travel* along the route r as

$$c_r := \sum_{(l,k)\in E_r} \rho_{lk} = \sum_{i=1}^{\theta_r} \sum_{(l,k)\in E_r^i} \rho_{lk} =: \sum_{i=1}^{\theta_r} c_r^i,$$

where c_r^i is the *travel cost per unit flow* on leg i of route r. We assume that the feeders on a route r start at o_r at the *last feasible start time*, i.e., at $(T - \sum_{(l,k)\in E_r} t_{lk})$. Thus, for each node l there is a unique *pickup time*, $t_r^i(l)$, for feeders on leg i of route r at which they pick up passengers at l.

B. Decision Variables and Constraints

Next, we discuss the decision variables and the constraints in the problem. We define f_r as *feeder volume on route* r, which is the total volume of vehicles sent on route r. We let $f_r^i(l)$ be the *allocation on node l in leg i of route* r, which is the volume of vehicles allocated to pick up passengers on node l in leg i of route r. We define $f_r(l)$ as the *allocation on a node l along route r* summed over all legs. *Total allocation on node l over all routes*, F_l, is

$$F_l := \sum_{r|l\in V_r} f_r(l) := \sum_{r|l\in V_r} \sum_{i|l\in V_r^i} f_r^i(l). \tag{4}$$

Note that the total volume of demand serviced at a node l can at most be d_l and hence may in general be less than F_l. Hence, we define $\tilde{f}_r^i(l)$ to be the *volume of passengers serviced* at a node l on leg i of route r, and $\tilde{F}_l$ as the *total volume of serviced passengers* at node l. Thus, f_r for each route r and $f_r^i(l)$ and $\tilde{f}_r^i(l)$ for each route r, leg i and node l are among the decision variables in our problem.

We now discuss the constraints in the problem, starting with allocation and supply constraints.

$$\sum_{l\in V_r^i} f_r^i(l) \le f_r, \quad \forall\, i \in [1, \theta_r]_{\mathbb{Z}}, \ \forall r \in \mathbf{R} \tag{5a}$$

$$\sum_{r|o_r=l} f_r \le S_l, \quad \forall\, l \in V \tag{5b}$$

The constraint (5a) is the *allocation constraint*, which ensures that the sum of all allocations in a leg i on a route r is at-most the route allocation f_r, while (5b) is the *supply constraint* that ensures that the sum of feeder volumes on all routes originating from node l, is at-most the supply S_l. Note that the flow conservation constraint is taken care-of as volumes on each route are defined separately. Hence we do not need to consider it explicitly. The constraints on passenger service variables are the following.

$$\tilde{f}_r^i(l) \le f_r^i(l), \quad \forall\, r, i, l \tag{6a}$$

$$\tilde{F}_l := \sum_{r,i,l} \tilde{f}_r^i(l) = \min\{F(l), d_l\}, \quad \forall l \in V \tag{6b}$$

C. Revenue, Cost and Profit Maximization

We next present the revenue and cost models. We define $p_r^i(l)$ as the *price per unit of service* at a node l, on leg i of route r. Each of these prices is an optimization variable constrained by $p_r^i(l) \ge 0$ and $p_r^i(l) \le \bar{p}_r^i(l)$. Here, $\bar{p}_r^i(l)$ is the *maximum viable price*, which is the maximum price that customers at node l are willing to pay to be serviced by leg i of route r. This maximum price depends on the alternate means of transportation available to customers at node l. We elaborate about pricing in the subsequent section.

The *revenues* at node l on leg i of route r are $p_r^i(l)\tilde{f}_r^i(l)$. We define the *total route expense* as $C_r := f_r c_r$, which can be interpreted as the fuel cost for the flow f_r on route r. Besides route costs, the service provider incurs *service costs* of F_l units, i.e. a unit cost per unit allocation on a node l, which can be understood as the cost of providing the service, commission/wages of the drivers, vehicle costs etc. Thus, *total costs* are $\sum_{r\in\mathbf{R}} C_r + \sum_{l\in V} F_l$.

With $p_r^i(l), f_r, f_r^i(l), \tilde{f}_r^i(l)$ as decision variables we define the service provider's optimization problem as

$$\begin{aligned} \max\ J := & \sum_{l\in V}\sum_{r\in\mathbf{R}}\sum_{i=1}^{\theta_r} p_r^i(l)\tilde{f}_r^i(l) - \sum_{r\in\mathbf{R}} C_r - \sum_{l\in V} F_l \\ \text{s.t.}\quad & (5),(6),\ f_r,\ f_r^i(l),\ \tilde{f}_r^i(l) \geq 0,\ p_r^i(l) \in [0, \bar{p}_r^i(l)] \\ & \forall\ r\in\mathbf{R},\ \forall\ i\in[1,\theta_r]_{\mathbb{Z}},\ \forall\ l\in V_r. \end{aligned} \tag{7}$$

Thus, J is the profit made by the service provider for serving the given demand in the localities defined by nodes under a given supply distribution. This being a non-linear problem with no restrictions on routing, it may be a computationally challenging task. Thus, in this paper, we broadly address three problems. The first problem is to obtain the optimal pricing, with which we reduce the problem to a linear program. The second problem we consider is that of providing an offline algorithm that would reduce the problem size. The third problem deals with an efficient way to compute the maximum profits for a given demand distribution among all supply distributions of a given total supply.

III. Optimal Pricing and Allocations

In this section, we first explore the issue of maximum viable price and then make some observations regarding the optimal pricing and allocations. Based on these observations, we simplify the optimization problem (7).

A. Maximum Viable Price and Optimal Price

The revenue from a node l on leg i of route r is a function of the price $p_r^i(l)$ that we charge a unit passenger for using the feeder service. The passenger has a choice to avail the service or to avoid it. In a competitive market, it is useful to identify $\bar{p}_r^i(l)$, the maximum viable price. For a price $p_r^i(l)$ to be viable, the resulting cost to a customer at node l must be lower than the best cost among the remaining means of transportation available to her. We model the cost of travel by a means of transportation as a combination of its monetary price and the value of travel time.

Suppose that for the best alternative means of transportation, the customers at a node l pay ζ_l units of money and it takes η_l units of time to commute to D. Let α be the value of time for the entire population in region G. Then the generalised travel cost by the best means of transportation from a node l to D is $g_l = \alpha\eta_l + \zeta_l$. Now for a leg i of route r passing through node l with pick-up time $t_r^i(l)$ a passenger travels effectively for $T - t_r^i(l)$ as the passenger has to wait at D before proceeding with the next mode of transport. Using this and the price $p_r^i(l)$, we can compute the generalised cost for node l along leg i of r, which for viability should be less than g_l, that is,

$$p_r^i(l) + \alpha(T - t_r^i(l)) \leq \alpha\eta_l + \zeta_l.$$

Hence, the maximum viable price is

$$\bar{p}_r^i(l) = -\alpha(T - t_r^i(l) - \eta_l) + \zeta_l.$$

Remark III.1. *(Optimal price is the maximum viable price). For any fixed $f_r^i(l)$, the total profit J is a strictly increasing function of $p_r^i(l)$. If the price $p_r^i(l) \leq \bar{p}_r^i(l)$, then it has no effect on any other constraints or on other optimization variables. Therefore, the optimal price $p_r^i(l) = \bar{p}_r^i(l)$.* •

B. Optimal Allocations

From the structure of the optimization problem (7) and as a consequence of Remark III.1, we can say that in any optimal solution, the allocations $f_r^i(l)$ and passengers served $\tilde{f}_r^i(l)$ would be the same.

Lemma III.2. *(Equivalence of optimal allocations and optimal volume of passengers served). In the model (7), for any optimal solution the allocations and passengers served are the same, that is $\tilde{f}_r^i(l) = f_r^i(l)$ and hence $\tilde{F}_l = F_l \leq d_l$.*

Given Lemma III.2, we use the terms *allocation at a node* and *service at a node* interchangeably. In addition, taking Remark III.1 into consideration, the original optimization problem (7) reduces to the following.

$$\begin{aligned} \max_{f_r, f_r^i(l)}\ \bar{J} := \sum_{l\in V}\sum_{r\in\mathbf{R}}\sum_{i=1}^{\theta_r} \beta_r^i(l) f_r^i(l) - \sum_{r\in\mathbf{R}} C_r \end{aligned} \tag{8}$$
s.t. (5), $F_l \leq d_l,\ f_r,\ f_r^i(l) \geq 0,\ \forall r\in\mathbf{R},\ \forall i,\ \forall l\in V,$

where $\beta_r^i(l)$ is the *allocation profitability*, and defined as

$$\beta_r^i(l) := \bar{p}_r^i(l) - 1. \tag{9}$$

IV. Properties of Optimal Solutions and Off-line Route Elimination

In this section, We discuss some important properties of the optimal solutions of the problem (8), which hold irrespective of the demand and supply distributions. With these properties we then go on to reduce the size of the problem by identifying and eliminating routes in $\mathbf{R}$ that will never be used in an optimal solution.

A. Properties of Optimal Solutions

We start by describing the cases where the constraint (5a) must be active. The following result states that the feeders on a route are allocated fully on each secondary leg.

Lemma IV.1. *(No redundant feeders in optimal solutions). In any optimal solution, the total allocation on a secondary leg of a route r is equal to the feeder volume on that route, f_r. That is, in any optimal solution,*

$$\sum_{l\in V_r^i} f_r^i(l) = f_r,\quad \forall i\in[2,\theta_r]_{\mathbb{Z}},\quad \forall r\in\mathbf{R}. \tag{10}$$

Next, we present necessary conditions for a route to have non-zero allocations in an optimal solution. These conditions follow from the K.K.T. conditions along with Lemma IV.1.

Proposition IV.2. *(Necessary conditions for a route to be used in any optimal solution). In any optimal solution, if $f_r > 0$ for $r \in \mathbf{R}$ then the following must necessarily hold.*

(a) $f_r^i(l) > 0$ for some $i \in [1, \theta_r]_{\mathbb{Z}}$ and $l \in V_r^i$. Further, for such (r, i, l), we must have $\beta_r^i(l) \geq 0$.

(b) The route r as a whole cannot make a loss, that is,

$$\sum_{i=1}^{\theta_r} \sum_{l \in V_r^i} f_r^i(l) \beta_r^i(l) \geq f_r c_r.$$

(c) For $i \in [2, \theta_r]_{\mathbb{Z}}$, $f_r^i(l) > 0$ only if $\beta_r^i(l) \geq c_r^i$. Further, for each $i \in [2, \theta_r]_{\mathbb{Z}}$, there must exist an $l \in V_r^i$ such that $\beta_r^i(l) \geq c_r^i$.

(d) If r is a simple route ($\theta_r = 1$), then $f_r^1(l) > 0$ implies $\beta_r^1(l) \geq c_r$. Further, there must exist atleast one $l \in V_r$ such that $\beta_r^1(l) \geq c_r$.

Proposition IV.2 says that in any optimal solution a route is utilized only if a profitable allocation can be made on the route. Further, in any optimal solution, a route is utilized only if there exists some node in each secondary leg or in the only leg of a simple route on which an allocation can fully offset the leg cost. Thus, Lemma IV.1 and Proposition IV.2 together can be used to eliminate routes and optimization variables that would never be utilized in any optimal solution. We detail such an algorithm in the next subsection.

B. Offline Route Elimination

Here we give Algorithm 1, which can be used to eliminate routes and optimization variables that would never be utilized in an optimal solution. The algorithm sweeps through each route in $\mathbf{R}$ and checks if it satisfies the necessary conditions on an optimal solution given in Lemma IV.1 and Proposition IV.2. If a route satisfies the necessary conditions then it is added to the *reduced route set* $\bar{\mathbf{R}}$. Since the necessary conditions in Lemma IV.1 and Proposition IV.2 hold independent of the demand and supply distributions, the reduced route set $\bar{\mathbf{R}}$ can be found offline.

Algorithm 1 : Offline route pruning algorithm

Input: $\mathbf{R}$
1: $\bar{\mathbf{R}} = \phi$
2: **for** $r \in \mathbf{R}$ **do**
3: **if** $\sum_{i=1}^{\theta_r} \sum_{l \in V_r^i | \beta_r^i(l) \geq 0} \beta_r^i(l) - c_r > 0$ **then**
4: **if** $\theta_r > 1$ and $\exists\, l \in V_r^i | \beta_r^i(l) \geq c_r^i \ \forall\, i \in (1, \theta_r]_{\mathbb{Z}}$ **then**
5: $\bar{\mathbf{R}} \leftarrow \bar{\mathbf{R}} \cup \{r\}$
6: **else if** $\exists\, l \in V_r^1 | \beta_r^1(l) \geq c_r^1$ **then**
7: $\bar{\mathbf{R}} \leftarrow \bar{\mathbf{R}} \cup \{r\}$
8: **end if**
9: **end if**
10: **end for**
Output: $\bar{\mathbf{R}}$

Here we explain the important steps of Alogrithm 1. In any optimal solution, a route r must satisfy Step 3 for (5a) and Proposition IV.2(b) to hold. Steps 4 and 5 are a direct application of Proposition IV.2(c) while Steps 6 and 7 enforce Proposition IV.2(d).

Lemma IV.1 and Proposition IV.2 guarantee that for any given demand and supply distributions, all optimal solutions use only the routes in the reduced route set $\bar{\mathbf{R}}$ from Algorithm 1. We formally state this fact in the following result.

Theorem IV.3. *(Optimal solutions use only the routes from the reduced route set). For the optimization problem (8), for any given demand and supply distributions, every optimal solution is guaranteed to have $f_r = 0$ and consequently $f_r^i(l) = 0$ over all legs i of r, $\forall\, r \notin \bar{\mathbf{R}}$.*

As a consequence of this result we can replace $\mathbf{R}$ with $\bar{\mathbf{R}}$ in the optimization problem (8) without affecting any optimal solutions or their values. The modified problem still remains a linear program. Thus, Algorithm 1 helps in reducing the number of optimization variables and thus saves on computational resources and memory.

V. Maximum Profits as a Function of Total Supply

In this section we analyse the problem of finding the maximum possible profits, for a fixed demand distribution, among all supply distributions with a given total supply. Based on this analysis, one can compute the maximum possible profits as a function of the total supply. Such bounds are helpful for studying economic viability, planning and resource allocation and possibly for load balancing.

The specific problem we are interested in solving is

$$\max_{S_l, f_r, f_r^i(l)} \bar{J} := \sum_{l \in V} \sum_{r \in \mathbf{R}} \sum_{i=1}^{\theta_r} \beta_r^i(l) f_r^i(l) - \sum_{r \in \mathbf{R}} C_r$$

$$\text{s.t. (5)}, \ F_l \leq d_l, \ f_r, \ f_r^i(l), \ S_l \geq 0, \ \forall r \in \mathbf{R}, \ \forall i, \ \forall l \in V,$$

$$\text{and } \sum_{l \in V} S_l = s. \tag{11}$$

Thus, compared to (8), here S_l is also as an optimization variable with the constraint that the total supply is equal to s. In addition, we make the following assumptions.

(A1) The demand distribution $\{(l, d_l) : l \in V\}$ is fixed.

(A2) For each node l in the graph $\exists\, r \in \bar{\mathbf{R}}$ s.t. $o_r = l$ and $\beta_r^1(l) - c_r^1 > 0$.

There is no loss of generality due to **(A2)**. This fact is made clear in the following proposition, which presents some other observations about all optimal solutions of problem (11).

Proposition V.1. *(Properties of optimal supply distributions and allocations). Every optimal solution to the problem (11) satisfies the following:*

*(a) If a node l does not satisfy **(A2)** then $f_r^i(l) = 0$ for all $r \in \mathbf{R}$ and all legs i of r.*

(b) For each $r \in \mathbf{R}$, $f_r^1(l) = 0, \ \forall l \neq o_r$ and $f_r^1(o_r) = f_r$.

(c) If $s < \sum_{l \in V} d_l$ then $\sum_{r | o_r = l} f_r = S_l, \ \forall l \in V$.

(d) If $s < \sum_{l \in V} d_l$ then $S_l \leq d_l, \ \forall l \in V$.

The implication of Proposition V.1(a) is that eliminating nodes l that do not satisfy the condition in **(A2)** does not change the overall solution or the maximum profits. Thus, there is no loss of generality in making the assumption, while it only simplifies some notation. As a consequence of Proposition V.1(b), we can say that in any optimal solution

$$\sum_{l \in V_r^i} f_r^i(l) = f_r, \quad \forall i \in [1, \theta_r]_{\mathbb{Z}}, \quad \forall r \in \mathbf{R}. \tag{12}$$

Thus, we can reduce (11) to an optimization problem over decision variables $f_r^i(l)$, the allocations and S_l, the supply at a node. This elimination of the variables f_r leads to a significant reduction in the number of optimization variables. Moreover, we can express the objective as

$$\max_{S_m,\ f_r^i(l)} \bar{J} = \sum_{l \in V_r^i} \sum_{r \in \bar{\mathbf{R}}} \sum_{i=1}^{\theta_r} (\beta_r^i(l) - c_r^i) f_r^i(l). \tag{13}$$

Now, we split further analysis into two cases, one with insufficient supply and the other with sufficient supply. In the latter case, we give the exact maximum profits among all supply distributions for the given demand distribution.

1) Maximum Profits with Insufficient Supplies: Using Proposition (V.1) and (12), we can solve (11) with strict equality in the constraints of (5a) and (5b). Thus, this is a much simpler problem to solve for a sequence of values of s than the original.

2) Maximum Profits with Sufficient Supply: We now study the case when overall supply is greater than overall demand. In the following theorem we give some properties of the optimal solution, including the optimal value. In the theorem, we use $\mathcal{R}(l)$, the set of simple routes originating at l and with the maximum rate of profits for a pickup at l, that is

$$\mathcal{R}(l) := \underset{r \in \bar{\mathbf{R}} | o_r = l,\ \theta_r = 1}{\text{argmax}} \{\beta_r^1(l) - c_r^1\}.$$

Theorem V.2. *If $s \geq \sum_l d_l$, then any optimal solution to (11) satisfies*

(a) $S_l \geq d_l,\ \forall\ l \in V$

(b) $F_l = \sum_{r | o_r = l} f_r^1(l) = d_l$ *s.t.* $f_r = f_r^1(o_r) \geq 0$ *and* $\theta_r = 1$

(c) If $r \notin \mathcal{R}(o_r)$ then $f_r = 0$. And the maximum possible profits over all supply distributions is

$$J_{max} = \sum_{l \in V} d_l \max_{r \in \bar{\mathbf{R}} | o_r = l,\ \theta_r = 1} \{\beta_r^1(l) - c_r^1\}. \tag{14}$$

This theorem gives the maximum possible profits, over all supply distributions, for a given demand distribution. It is only a function of the demand distribution and the graph parameters. All the optimal solutions with sufficient supply only use simple routes.

VI. Simulation Results

Since (8) is a linear program, we utilize disciplined convex optimization [25] based CVXpy [26] for simulations. We performed simulations on the 20 node graph given in Figure 1. The destination is node 19 and the time to reach the destination is $T = 20$.

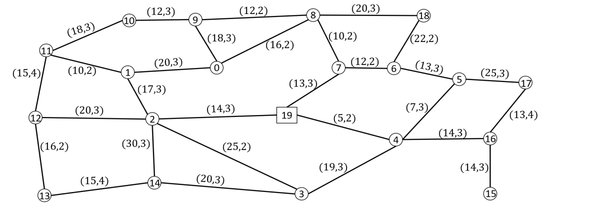


Fig. 1: Simulation graph: The numbers in the circle represents node index l, the edge weights tuple (a, b) represent (ρ_{lk}, t_{lk}) and Destination, $D = 19$ is marked in square

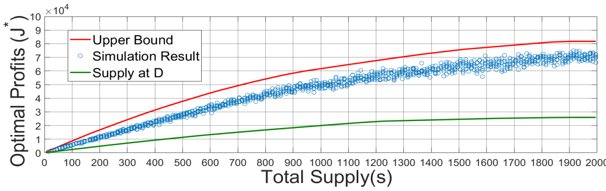
To ensure η_l and ζ_l are comparable to the network we assume that the best alternative transport available in the region costs bc_r for a route $r \in \bar{\mathbf{R}}$. Then calculating the generalised cost for a route p, $g_p = \alpha t_p + c_p$ we let the best available routes to passengers at node l as $r(l)^*$ with

$$\begin{aligned} r(l)^* &\in \underset{r \in \mathcal{R}(l)}{\text{argmin}} \{\alpha t_r + bc_r\} \\ \eta_l &= t_{r(l)^*}, \quad \zeta_l = bc_{r(l)^*}. \end{aligned} \tag{15}$$

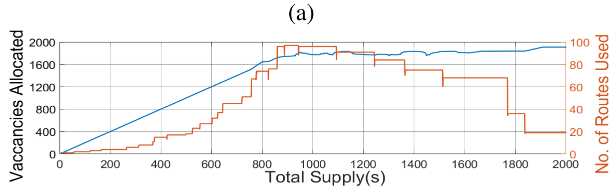
Next, Figure 2 demonstrates a simulation of the various aspects of the problem with value of time assumed as $\alpha = 0.5$ units, and the value of b in (15) assumed as 2.5. The number of feasible routes in $\mathbf{R}$ is 8732 as per definition (3) and the number of routes in $\bar{\mathbf{R}}$ is 3075 based on the application of Algorithm 1. The number of optimization variables with $\mathbf{R}$ is 48495 but with $\bar{\mathbf{R}}$ it is 17187 which is approximately a third of the ones in the original problem. For finding the max profits with insufficient supply, the formulation in (13) has 14132 number of variables.

We then set a fixed demand profile where d_l, $l \in V \setminus \{D\}$, was drawn randomly and independently from a uniform distribution over $[0, 150]_{\mathbb{Z}}$. The total demand was $\sum_{l \in V} d_l = 1910$. We generate the optimal profits as a function of total supply using the objective in (13) and constraints described in Section V-.1 with total supply s chosen with a spacing of 10 in $[0, 1905]_{\mathbb{Z}}$. Beyond $s = 1905$ we utilise (11) to validate Theorem V.2, which produces a maxima of 81730.0 based on (14) which is within numerical error of the maxima value calculated by CVXpy. The scatter points in Figure (2a) each represent a simulation with a different randomly chosen supply distribution with S_l, $l \in V$ drawn from a independent uniform distribution in $[0, 150]$. The case where supply is concentrated at the destination only is marked by a green line in Figure (2a). Though, it is found to do the worst amongst all simulations but is not proven to be a strict lower bound.

Figure (2b) presents the number of routes used and the total vacancies allocated with respect to total supply s for an optimal solution to (13) and constraints listed in Sections V-.1, and V-.2. There can in general be multiple solutions with different properties. It can be noted that beyond a certain total supply the optimal solutions using simple routes increases and the number of routes used for generating upper-bound drops down to 19 simple routes belonging to $\mathcal{R}(l)$, $\forall\ l \in V \setminus \{D\}$ in correspondence to Theorem V.2.



(a)



(b)

Fig. 2: Simulation Results: **(a):** Simulations Results: Maximum profits vs total supply in Red. Profits for distribution $S_D = s$ in green. Different supply distributions as blue scatter points. **(b):** No. of used routes and Vacancies allocated in obtaining the maximum profits for a best distribution

VII. Conclusions

In this paper we propose a *one-shot* optimization problem for single-event single-destination fixed time horizon feeding for coordinating first-mile pick-ups and destination drop-offs in multi-modal transportation. We pose the problem as one of maximization of profits and thus is from the service provider's point of view. We analyze the pricing aspect which is useful for viability studies. Then, based on the necessary properties of the optimal solutions, we provide an off-line algorithm that does not depend on the knowledge of the demand or supply distributions and reduces the problem size significantly. We also provide a computational method for obtaining the maximum possible profits for a given demand distribution as a function of the total supply. We validate our analysis through simulations in multiple scenarios with fixed demand for a one-shot coordination.

As this is a one-shot deterministic demand matching problem, therefore an extension of this problem is redistribution of supply with some demand anticipative scheme. Sensitivity and robustness analysis of the performance of such a feeding scheme can be tested against various degrees of mismatch of anticipative data and actual demand data which may be useful in planning and viability of the existing scheme.

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