Event-Triggered Parameterized Control for Stabilization of Linear Systems

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Abstract— **This paper proposes a new control method called event-triggered parameterized control (ETPC). We showcase this method by focusing on the specific problem of stabilization of linear systems. In this control method, between two consecutive events, each control input to the plant is a linear combination of a set of linearly independent scalar functions. At each event, the coefficients of the parameterized control input are chosen to minimize the error in approximating a model based control signal and then they are communicated to the actuator. We design two event-triggering rules that guarantee global asymptotic stability of the origin of the closed loop system under some conditions on the model uncertainty. We also show the existence of a uniform positive lower bound on the inter-event times. We illustrate our results through numerical examples. We compare the proposed control method with event-triggered zero-order-hold control and show a significant improvement in terms of the average inter-event times.**

I. INTRODUCTION

Event-triggered control is a popular method in the field of networked control systems owing to its advantage of efficient utilization of resources for a large variety of systems and control objectives. Most of the existing event-triggered controllers are designed using sampled-data zero-order-hold (ZOH) control input. However, many of the communication protocols used in networked control systems, such as TCP and UDP, have a minimum packet size [1]. So, ZOH control may lead to under utilization of each packet while also increasing the number of communication instances and hence packets. On the other hand, use of non-ZOH control leads to better utilization of the minimum payload of each packet while also reducing the overall number of events or communication instances. With these motivations, in this paper, we propose an event-triggered parameterized control method for stabilization of linear systems. While model based event-triggered control or deadbeat control or first-order hold control share our motivations, our approach differs significantly from them and also has several advantages over them. We discuss more about this in Section I-B.

A. Literature Review

Event-triggered control literature is very broad, and a quick introduction to the topic can be found in [2]–[5]. Modified event-triggered control methods, such as self-triggered control [6] and periodic event-triggered control [7], are also equally popular as they provide some practical advantages over plain event-triggered control. In all these methods, typically the control input to the plant is held constant between two events.

A major exception to this rule is model-based event-triggered control [8]–[12], where the control input applied to the plant is time-varying even between two consecutive events and is generated by a model of the system. Essentially, both the controller and the actuator have identical copies of the model and synchronously update the state of the model in an event-triggered manner.

In event/self-triggered model predictive control, see for example [13]–[15], a control trajectory is generated by solving a finite horizon optimal control problem at each triggering instant. The actuator applies this optimal control trajectory to the plant until it receives a new control trajectory at the next triggering instant. A few recent papers [16], [17] show that usage of communication resources can be further reduced by communicating only some of the samples of the computed control trajectory to the actuator. Then the actuator applies the control input to the plant based on first-order-hold (FOH) between the samples. [18] takes this idea further and designs adaptive selection of samples under this control method.

The literature on event-triggered control based on non-ZOH also includes the paper [19] which deals with event-triggered dead-beat control. This paper considers a stochastic system where the controller transmits a sequence of control inputs to the actuator over an unreliable communication channel in an event-triggered manner. This control sequence is stored in a buffer and applied sequentially until the next control packet arrives.

Another control method, in the distributed event-triggered control setting that shares similar motivation as in our paper is the team triggered control method [20], [21]. In this control method, agents make promises to their neighbors about their future time-varying states or controls and inform them if these promises are violated later.

B. Contributions

The major contribution of this paper is a new control method called event-triggered parameterized control (ETPC). We showcase the method for the task of stabilization of linear systems. In this method, between two consecutive events, each control input to the plant is a linear combination of a fixed set of linearly independent scalar functions. At each event the coefficients of the parameterized control input are chosen so as to minimize the error in approximating a model based control signal. The parameters of the control

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input so computed are communicated to the actuator in an event-triggered manner. In this paper, we design two event-triggering rules that guarantee global asymptotic stability of the origin of the closed loop system under some bounds on the modeling error. We also guarantee a positive lower bound on the inter-event times generated by both the rules.

Our approach generalizes both ZOH and FOH control. In addition, our approach also allows the applied control input to differ from the ideal emulated control input even at the event times. Compared to ZOH or even FOH control, our method can be fine tuned to utilize the full payload per each packet of communication and as a result also requires fewer communication instances or packets.

Model-based event-triggered control requires the actuator to have enough computational resources to simulate the model online. This can be particularly difficult if the system is nonlinear or the control task is complicated. In addition, the updation of the state of the model has to happen synchronously at both the controller and the actuator. If there are unknown time delays in communication, then this can be a challenge. If one also carries out dynamic quantization along with model based event-triggered control, such as in [22]–[24], then unknown time delays can also cause desynchronization of the dynamic quantization frame, which is a very severe issue. Finally, sharing a copy of the model (and inherently the control goal) of the system at the actuator or at multiple nodes in a distributed control application is very undesirable from a privacy and security point of view.

In methods as in event-triggered MPC or deadbeat control or the sampling approach in [16]–[18] essentially require a large number of samples to be communicated, at each event, to achieve the same level of approximation of an ideal control signal as achieved by our proposed parametrized control approach. This is because, our proposed method requires only a limited number of parameters to be sent irrespective of the time duration of the signal. Moreover, event-triggered MPC or deadbeat control frameworks have primarily been explored in a discrete-time setting. Our approach is not restricted to discrete-time settings.

In summary, our proposed approach has the following advantages: a time-varying control input between events, the complexity of which can be tuned; better utilizes the communication resources; requires lesser computational resources at the actuators; does not suffer from synchronization issues; and can be easily generalized to a variety of problem settings; and provides greater privacy and security than model based control methods.

C. Organization

Section II formally presents the system dynamics and the objective of this paper. In Section III, we design a control law and two event-triggering rules to achieve our objective. Then, we analyze the proposed controllers and show global asymptotic stability of the origin of the closed loop system as well as non-Zeno behavior of inter-event times. Section IV illustrates the results using numerical examples. Finally, we provide some concluding remarks in Section V.

D. Notation

Let $\mathbb{R}$ denote the set of all real numbers. Let $\mathbb{N}$ and $\mathbb{N}_0$ denote the set of all positive and non-negative integers, respectively. For any $x \in \mathbb{R}^n$, $\|x\|$ denotes the euclidean norm. For an $n \times n$ square symmetric matrix A, let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the smallest and the largest eigenvalues of A, respectively. For an $n \times m$ matrix B, let B_i denote the i^{th} row of B. For any two functions $v, w : [0,T] \to \mathbb{R}$, let

$$\langle v, w \rangle := \int_0^T v(\tau) w(\tau) \mathrm{d}\tau.$$

II. PROBLEM SETUP

In this section, we present the system dynamics and set up the problem that we are interested in.

System Dynamics and Control Input

Consider a linear time-invariant system,

$$\dot{x} = Ax + Bu, \quad \forall t \geq t_0, \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$, respectively, denote the system state and the control input. $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the system matrices. Let $\hat{A}$ and $\hat{B}$ denote the available model of A and B, respectively. We assume that there exists a $K \in \mathbb{R}^{m \times n}$ such that $\hat{A}_c := \hat{A} + \hat{B}K$ is Hurwitz.

Suppose we wish to control the given system with event-triggered communication of actuation signals. However, compared to most of the literature on event-triggered control, we seek to apply a time-varying input between two consecutive events. In order to still be able to transmit limited information over the network, we consider time-varying control inputs in a space of functions with finitely many parameters. In particular, we let the i^{th} control input, for $i \in \{1, 2, \ldots, m\}$, during an inter-event time be

$$u_i(t_k + \tau) = f(\mathbf{a}_i(k), \tau) := \sum_{j=0}^{p} a_{ij}(k) \phi_j(\tau), \ \forall \tau \in [0, t_{k+1} - t_k). \tag{2}$$

Here $(t_k)_{k \in \mathbb{N}_0}$ is the sequence of communication time instants from the controller to the actuator. At t_k, the controller communicates the coefficients of the parameterized control input, $\mathbf{a}(k) := [a_{ij}(k)] \in \mathbb{R}^{m \times (p+1)}$, to the actuator. We also let $\mathbf{a}_i(k)$ denote the i^{th} row of $\mathbf{a}(k)$. Finally,

$$\Phi := \{\phi_j : [0, T] \to \mathbb{R}\}_{j=0}^{p}$$

is a set of fixed functions on the time interval $[0,T] \subset \mathbb{R}_{\geq 0}$ for some fixed finite $T > 0$. We make the following standing assumption about Φ through out this paper.

- **(A1)** Each function $\phi_j \in \Phi$ is continuously differentiable. ϕ_0 is a non-zero constant function and $\phi_j(0) = 0, \quad \forall j \in \{1, 2, \ldots, p\}$. Φ is a set of linearly independent functions.

By linearly independent functions, we mean that

$$\sum_{j=0}^{p} c_j \phi_j(t) = 0, \quad \forall t \in [0, T] \ \text{ iff } c_j = 0, \ \forall j \in \{0, 1, \ldots, p\}.$$

Figure 1 represents the general configuration of the event-triggered parameterized control system that we consider in this paper. Note that, here, the communication time instants from the controller to the actuator are determined by an event-triggering rule depending on the system state and the control input that are available to the triggering mechanism continuously.

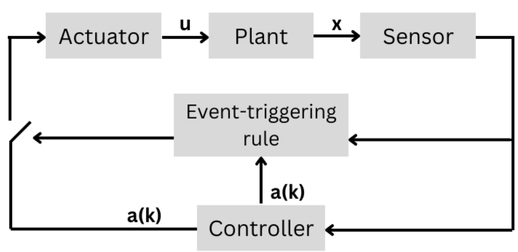


Fig. 1: Event-triggered parameterized control configuration

Objective

Our objective is to design a parameterized control law (2) and a triggering rule for implicitly determining the communication times $(t_k)_{k\in\mathbb{N}_0}$, at which the parameters of the controller are updated so that the origin of the closed loop system is globally asymptotically stable. We also wish to ensure that the inter-event times in the closed loop system have a uniform positive lower bound.

III. Design and Analysis of Event-Triggered Controller

In this section, we first design a control law and two event-triggering rules to achieve our objective. Later, we analyze the performance of the designed event-triggered control system.

A. Control Law

The idea behind the proposed control law is to emulate an ideal open loop control signal using a parametrized time-varying signal as in (2). Since the goal here is asymptotic stabilization of the origin of the closed loop system, consider the "ideal" model based control signal $K\hat{x}(t_k+\tau)$, where

$$\dot{\hat{x}} = \hat{A}_c\hat{x}, \quad \forall \tau \in [0,T], \quad \hat{x}(t_k) = x(t_k),\ k \in \mathbb{N}_0. \tag{3}$$

However, sending the complete control signal $K\hat{x}(t_k+\tau)$ for $\tau \in [0,T)$ in a communication packet at t_k is not possible without approximation. We thus find the best fit for the function $K\hat{x}(t)$ in the linear span of Φ. In particular, we choose the coefficients of the parameterized control signal starting at t_k by solving the following finite horizon optimization problems, for $i \in \{1,2,\dots,m\}$,

$$\begin{aligned} \mathbf{a}_i(k) = \operatorname*{arg\,min}_{a\in\mathbb{R}^{1\times(p+1)}} & \int_0^T |f(a,\tau) - K_i\hat{x}(t_k+\tau)|^2 d\tau, \\ \text{s.t.} \quad & |f(a,0) - K_i\hat{x}(t_k)| \le \varepsilon \, \|\hat{x}(t_k)\| \end{aligned} \tag{4}$$

for some $\varepsilon \ge 0$ and a finite time horizon $T > 0$ which are to be designed. K_i denotes the i^{th} row of K for $i \in \{1,2,\dots,m\}$.

Note that, in order to solve the optimization problem (4), we require the signal $K\hat{x}(t_k+\tau)$ for $\tau \in [0,T]$. This signal may be obtained directly as $Ke^{\hat{A}_c\tau}\hat{x}(t_k)$ or through numerical simulation of the $\hat{x}$ dynamics (3). The latter method is generalizable to nonlinear systems or to tasks other than asymptotic stabilization in linear systems.

Remark 1. *(Control signal for $\tau > T$). Given the parameters $\boldsymbol{a}(k)$ that are obtained by solving (4), the control signal that is applied by the actuator is as given in (2). However, since t_k's are implicitly determined by an event-triggering rule online, it may happen that $t_{k+1} - t_k > T$. In that case, we simply extend the control input $u(t_k+\tau)$ for τ beyond T by suitably extending the domain of the functions $\phi_j \in \Phi$.* •

Remark 2. *(Feasibility of (4) and comparison with zero-order hold control). Under Assumption **(A1)**, there is a non-zero constant function $\phi_0 \in \Phi$. Hence, there exists some a such that $f(a,\tau)$ is the zero-order hold signal $K_i\hat{x}(t_k)$ for all $\tau \in [0,T]$, which is feasible. Note that if $\varepsilon > 0$ then zero-order hold or constant controls other than $K_i\hat{x}(t_k)$ for all $\tau \in [0,T]$ are also feasible. Indeed in general, for the control signal $u_i(.)$, as in (2), that we obtain through solving (4) need not have $u_i(t_k) = K_i\hat{x}(t_k)$, which is typically not the case in emulation based event-triggered controllers.* •

Proposition 3. *The optimization problem (4) is a convex optimization problem.*

Proof. Let us first show that the objective function of (4) is convex. Note that, as the integral of a convex function is also convex, it is sufficient to show that $\bar{f} := |f(a,\tau) - K_i\hat{x}(t_k+\tau)|^2$ is convex in the optimization variables a. The Hessian matrix of the above function, $\mathbf{H}_{\bar{f}}$, is

$$\mathbf{H}_{\bar{f}} = 2\begin{bmatrix} \phi_0^2(\tau) & \phi_0(\tau)\phi_1(\tau) & \dots & \phi_0(\tau)\phi_p(\tau) \\ \phi_1(\tau)\phi_0(\tau) & \phi_1^2(\tau) & \dots & \phi_1(\tau)\phi_p(\tau) \\ \dots & \dots & \dots & \dots \\ \phi_p(\tau)\phi_0(\tau) & \phi_p(\tau)\phi_1(\tau) & \dots & \phi_p^2(\tau) \end{bmatrix}.$$

Observe that this matrix has eigenvalues at $2\sum_{j=0}^{p}\phi_j^2(\tau)$ and at 0, with algebraic multiplicity p. Thus, $\mathbf{H}_{\bar{f}}$ is positive semi-definite. Hence, the cost function in (4) is convex. Note that under Assumption **(A1)**, the only constraint in the optimization problem (4) can be written as two linear constraints in a_0, the coefficient of the constant function in Φ. Thus, (4) is a convex optimization problem. □

B. Event-Triggering Rule

Consider the candidate Lyapunov function $V(x) = x^T Px$, where $P > 0$ and satisfies $\hat{A}_c^T P + P\hat{A}_c = -Q$ for some $Q > 0$. Such a P exists as $\hat{A}_c$ is Hurwitz. We consider two event-triggering rules. The first one is

$$t_{k+1} = \min\{t > t_k \ : \ \|q\| \ge \sigma\,\|x\|\},\ q(t) := u(t) - Kx(t), \tag{5}$$

where $\sigma > 0$ is a design parameter. The second event-triggering rule is

$$t_{k+1} = \min\{t > t_k \ : V(x(t)) \geq V(x(t_k))\mathrm{e}^{-rh\tau}\}, \quad (6)$$

where $\tau = t - t_k$, $h = \dfrac{\lambda_{min}(Q)}{\lambda_{max}(P)}$ and $r \in (0,1)$ is a design parameter.

C. Analysis of the event-triggered control system

Next, we show that the origin of the closed loop system (1) under the control law (2)-(4) and either of the event-triggering rules (5), or (6), is globally asymptotically stable and show the existence of a uniform positive lower bound on the inter-event times. Let us first analyze the closed loop system under the event-triggering rule (5).

Theorem 4. *(Global asymptotic stability and absence of Zeno behavior under the event-triggering rule (5)). Consider the system (1) under the parametrized control law (2)-(4) and the event-triggering rule (5). Suppose that the modeling error $\tilde{A}_c := \hat{A}_c - (A+BK)$ satisfies $\|\tilde{A}_c\| \leq \dfrac{\rho\lambda_{\min}(Q)}{4\|P\|}$, that in (4), $\varepsilon < \dfrac{\sigma}{2\sqrt{m}}\sqrt{\dfrac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$ and that $\sigma \leq \frac{\rho\lambda_{\min}(Q)}{4\|PB\|}$ for some $\rho \in (0,1)$. Then, the origin of the closed loop system is globally asymptotically stable and there exists a uniform positive lower bound on the inter-event times generated by the closed loop system.*

Proof. Let us first calculate the time derivative of the candidate Lyapunov function along the trajectories of the closed loop system.

$$\begin{aligned}
\dot{V}(x) &= -x^T Qx + 2x^T P(Bq - \tilde{A}_c x) \\
&\leq -\lambda_{\min}(Q)\|x\|^2 + 2\|x\|\|PB\|\|q\| + 2\|x\|^2\|P\|\|\tilde{A}_c\| \\
&\leq -(1-\rho)\lambda_{\min}(Q)\|x\|^2 - \left(\frac{\rho}{2}\lambda_{\min}(Q) - 2\sigma\|PB\|\right)\|x\|^2 \\
&\quad - \left(\frac{\rho}{2}\lambda_{\min}(Q) - 2\|P\|\|\tilde{A}_c\|\right)\|x\|^2 \\
&\leq -(1-\rho)\lambda_{\min}(Q)\|x\|^2.
\end{aligned}$$

In the second inequality, we have used the fact that the event-triggering rule (5) ensures $\|q\| \leq \sigma\|x\|$. The last inequality follows from the conditions on $\|\tilde{A}_c\|$ and σ in the statement of the result. Thus, the origin of the closed loop system is globally asymptotically stable.

Next, let us show that the inter-event times do not exhibit Zeno behavior and that in fact they have a uniform positive lower bound by using the following claims.

Claim (a): There exist $\beta_1, \beta_2 > 0$ such that $\|u(t)\| \leq \beta_1\|x(t_k)\|$ and $\|\dot{u}(t)\| \leq \beta_2\|x(t_k)\|$, $\forall t \in [t_k, \min\{t_{k+1}, t_k + T\})$, $\forall k \in \mathbb{N}_0$.

Claim (b): There exists a monotonically decreasing function $g : \mathbb{R} \to \mathbb{R}$ such that $\|x(t)\|^2 \geq g(t-t_k)\|x(t_k)\|^2$ for all $t \in [t_k, t_{k+1})$ with $g(0) = \gamma > 0$ for all $k \in \mathbb{N}_0$.

Let us first prove claim (a). Note that, $\forall i \in \{1,2,\ldots,m\}$ and for any $k \in \mathbb{N}_0$, $u_i(t)$ for $t \in [t_k, t_{k+1})$ is chosen by solving the optimization problem (4). That is, $u_i(t_k+\tau)$ is the continuous least squares approximation of $\hat{u}_i(\tau) := K_i\hat{x}(t_k+\tau)$ with linear constraints. We can rewrite the optimization problem (4) as follows

$$\min_{a \in \mathbb{R}^{1\times(p+1)}} E_i(a), \quad \text{s.t. } F_i(a) \leq 0,$$

where,

$$E_i(a) = \langle \hat{u}_i, \hat{u}_i \rangle - 2\sum_{j=0}^{p} a_j \langle \hat{u}_i, \phi_j \rangle + \sum_{j=0}^{p}\sum_{l=0}^{p} a_j a_l \langle \phi_j, \phi_l \rangle,$$

$$F_i(a) = \begin{bmatrix} a_0\phi_0(0) - \hat{u}_i(0) - \varepsilon\|\hat{x}(t_k)\| \\ -a_0\phi_0(0) + \hat{u}_i(0) - \varepsilon\|\hat{x}(t_k)\| \end{bmatrix}.$$

Now, we can write the corresponding Lagrangian as $\mathcal{L}_i(a,\mu) = E_i(a) + \mu^T F_i(a)$, where $\mu \in \mathbb{R}^2$ is the Lagrange multiplier vector. Recall from Proposition 3 that the optimization problem (4) is convex and the constraints $F_i(a) \leq 0$ are linear in a. So, strong duality holds for the problem (4). So, an optimal primal-dual solution $(\mathbf{a}_i(k), \mu_i(k))$ must satisfy the Karush-Kuhn-Tucker (KKT) conditions. The stationarity conditions can be represented in matrix form as follows,

$$G\mathbf{a}_i^T(k) = D_i(k)$$

where

$$G = 2\begin{bmatrix} \langle\phi_0,\phi_0\rangle & \langle\phi_0,\phi_1\rangle & \ldots & \langle\phi_0,\phi_p\rangle \\ \langle\phi_1,\phi_0\rangle & \langle\phi_1,\phi_1\rangle & \ldots & \langle\phi_1,\phi_p\rangle \\ \ldots & \ldots & \ldots & \ldots \\ \langle\phi_p,\phi_0\rangle & \langle\phi_p,\phi_1\rangle & \ldots & \langle\phi_p,\phi_p\rangle \end{bmatrix},$$

$$D_i(k) = 2\begin{bmatrix} \langle\hat{u}_i,\phi_0\rangle \\ \langle\hat{u}_i,\phi_1\rangle \\ \ldots \\ \langle\hat{u}_i,\phi_p\rangle \end{bmatrix} - \begin{bmatrix} \phi_0(0) & -\phi_0(0) \\ 0 & 0 \\ \ldots & \ldots \\ 0 & 0 \end{bmatrix}\mu_i(k).$$

Further, the complementary slackness conditions are

$$\begin{aligned}
\mu_{i1}(k)(\mathbf{a}_{i0}(k)\phi_0(0) - \hat{u}_i(0) - \varepsilon\|\hat{x}(t_k)\|) = 0 \\
\mu_{i2}(k)(-\mathbf{a}_{i0}(k)\phi_0(0) + \hat{u}_i(0) - \varepsilon\|\hat{x}(t_k)\|) = 0.
\end{aligned}$$

Now, observe that G is twice the Gram matrix for the functions in Φ. Since the set of functions Φ are linearly independent, we can say that G is invertible. So, every optimal primal-dual solution $(\mathbf{a}_i(k), \mu_i(k))$ to the problem (4) must satisfy $\mathbf{a}_i^T(k) = G^{-1}D_i(k)$. Next, note that

$$\hat{u}_i(\tau) = K_i\hat{x}(t_k+\tau) = K_i\mathrm{e}^{\hat{A}_c\tau}\hat{x}(t_k),$$

where the second equality follows from (3). Thus, the first term in $D_i(k)$ is a constant matrix times $\hat{x}(t_k)$. If for the optimal solution $\mathbf{a}_i(k)$ to problem (4), the constraints are not active then $\mu_i(k) = 0$ and we can see that each $|\mathbf{a}_i(k)|$ is upper bounded by a constant factor times norm of $\hat{x}(t_k)$. If on the other hand, $\mu_i(k) \neq 0$, then $\mathbf{a}_{i0}(k)$ can be solved for from the complementary slackness conditions and its magnitude is upper bounded by some constant times norm of $\hat{x}(t_k)$. By using the last p stationarity conditions, we can also show that the magnitude of $\begin{bmatrix}\mathbf{a}_{i1}(k) & \ldots & \mathbf{a}_{ip}(k)\end{bmatrix}$ is upper bounded by some constant times norm of $\hat{x}(t_k)$. This implies that each

$|\mathbf{a}_i(k)|$ is upper bounded by a constant factor times norm of $\hat{x}(t_k) = x(t_k)$. So, we can say that there exists $\beta' > 0$ such that $\|\mathbf{a}(k)\| \leq \beta' \|x(t_k)\|$, $\forall k \in \mathbb{N}_0$. Since each $\phi_j(.) \in \Phi$ is continuously differentiable on $[0,T]$, we can say that there exist $\beta_1, \beta_2 > 0$ such that $\forall t \in [t_k, \min\{t_{k+1}, t_k+T\})$, $\forall k \in \mathbb{N}_0$,

$$\|u(t)\| \leq \|\mathbf{a}(k)\| \left\| \begin{bmatrix} \phi_0(t-t_k) \\ \vdots \\ \phi_p(t-t_k) \end{bmatrix} \right\| \leq \beta_1 \|x(t_k)\|,$$

$$\|\dot{u}(t)\| \leq \|\mathbf{a}(k)\| \left\| \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \phi_0(t-t_k) \\ \vdots \\ \phi_p(t-t_k) \end{bmatrix} \right\| \leq \beta_2 \|x(t_k)\|.$$

This proves claim (a).

Next, let us prove claim (b). Note that, as $V(x(t))$ is a monotonically decreasing function of t, $\|x(t)\| \leq c_1 \|x(t_k)\|$ where $c_1 := \sqrt{\dfrac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ for all $t \geq t_k$. For all $t \in [t_k, t_{k+1})$,

$$\dot{V}(x) \geq -x^T Q x - \left\|2x^T P(Bq - \tilde{A}_c x)\right\| \geq -c_2 V(x) - c_3 \|x(t_k)\|^2,$$

where $c_2 = \frac{\lambda_{\max}(Q) + 2\|P(BK+\tilde{A}_c)\|}{\lambda_{\min}(P)}$ and $c_3 = 2c_1\beta_1 \|PB\|$. Now, by using comparison lemma, we can show that,

$$V(x(t)) \geq \mathrm{e}^{-c_2(t-t_k)} V(x(t_k)) + \frac{c_3}{c_2}(\mathrm{e}^{-c_2(t-t_k)} - 1) \|x(t_k)\|^2.$$

This implies that claim (b) is true with,

$$g(\tau) := \frac{1}{\lambda_{\max}(P)} \left[\mathrm{e}^{-c_2\tau} \left(\lambda_{\min}(P) + \frac{c_3}{c_2} \right) - \frac{c_3}{c_2} \right]$$

and $g(0) = \gamma := \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)} > 0$ for all $k \in \mathbb{N}_0$.

Next, note that the choice of $\varepsilon < \dfrac{\sigma}{2\sqrt{m}} \sqrt{\dfrac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$ ensures that $\|q(t_k)\|^2 < \dfrac{\sigma^2\gamma}{4} \|x(t_k)\|^2$. Now, let T_1 denote the time it takes $g(.)$ to decrease from γ to $(\gamma/2)$ and let T_2 denote the time it takes $\|q\|^2$ to grow from $\dfrac{\sigma^2\gamma}{4}\|x(t_k)\|^2$ to $\dfrac{\sigma^2\gamma}{2}\|x(t_k)\|^2$. Then, the inter-event times are lower bounded by the minimum of T_1 and T_2.

Now, to show that the lower bound on the inter-event times is uniform, first note that T_1 is independent of $x(t_k)$. Next, let us analyze the time derivative of $\|q\|^2$.

$$\begin{aligned} \frac{d}{dt}\|q\|^2 &= 2q^T\dot{q} = 2q^T(\dot{u} - K(Ax+Bu)) \\ &\leq 2\|u - Kx\| \left[\|\dot{u}\| + \|KA\|\|x\| + \|KB\|\|u\| \right] \\ &\leq \beta \|x(t_k)\|^2, \end{aligned}$$

for some $\beta > 0$. The last inequality follows from claim (a) and the fact that $\|x(t)\| \leq c_1 \|x(t_k)\|$ for all $t \geq t_k$. This implies that $T_2 \geq \dfrac{\sigma^2\gamma}{4\beta}$, which completes the proof of this result. □

Note that the upper bound on σ given in Theorem 4 depends on the norm of the input matrix B which is unknown. But, in practice, we can use some known upper bound on $\|B\|$ to choose the value of σ. Such an upper bound can also come from the known model $\hat{B}$ and bounds on the modeling error $B - \hat{B}$.

Next, we analyze the performance of the closed loop system under the event-triggering rule (6).

Theorem 5. *(Global asymptotic stability and absence of Zeno behavior under the event-triggering rule (6)). Consider the system (1) under the parametrized control law (2)-(4) and the event-triggering rule (6). Suppose that the modeling error $\tilde{A}_c := \hat{A}_c - (A+BK)$ satisfies $\|\tilde{A}_c\| \leq \dfrac{\rho\lambda_{\min}(Q)}{4\|P\|}$, that in (4), $\varepsilon < \dfrac{\sigma}{2\sqrt{m}}\sqrt{\dfrac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$ and that $\sigma \leq \dfrac{\rho\lambda_{\min}(Q)}{4\|PB\|}$ for some $\rho \in (0,1)$. Also, suppose that in (6), $r \in (0,(1-\rho))$. Then, the origin of the closed loop system is globally asymptotically stable and there exists a uniform positive lower bound on the inter-event times generated by the event-triggering rule (6).*

Proof. First note that, the triggering rule (6) guarantees that

$$V(x(t)) \leq V(x(t_k))\mathrm{e}^{-rh(t-t_k)}, \quad \forall t \in [t_k, t_{k+1}),$$

for all $k \in \mathbb{N}_0$. If the sequence of inter-event times does not exhibit Zeno behavior, then we can say that

$$V(x(t)) \leq V(x(t_0))\mathrm{e}^{-rh(t-t_0)}, \quad \forall t \geq t_0.$$

Thus, the origin of the closed loop system is globally asymptotically stable.

Next, let us show that there exists a uniform positive lower bound on the inter-event times by using the following claim.

Claim (c): Let $\tau_1(x)$ and $\tau_2(x)$, respectively, denote the inter-event time functions corresponding to the event-triggering rules (5) and (6), respectively. In particular, $t_{k+1} = t_k + \tau_i(x(t_k))$, for each of the two rules. Under the given conditions on r, ε and $\|\tilde{A}_c\|$, $\tau_1(x) \leq \tau_2(x)$, $\forall x \in \mathbb{R}^n$.

Let us prove claim (c). Let $x(t_k) = x_k$ for some $x_k \in \mathbb{R}^n$ and $k \in \mathbb{N}_0$. From the proof of Theorem 4, we see that the event-triggering rule (5) ensures that

$$\dot{V}(x(t)) \leq -(1-\rho)hV(x(t)), \quad \forall t \in [t_k, t_k + \tau_1(x_k)),$$

where $h := \dfrac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$. This implies that,

$$V(x(t)) \leq V(x_k)\mathrm{e}^{-(1-\rho)h\tau}, \quad \forall t \in [t_k, t_k + \tau_1(x_k)),$$

where $\tau = t - t_k$. Thus, for all t in the open interval $(t_k, t_k + \tau_1(x_k))$,

$$V(x(t)) - V(x_k)\mathrm{e}^{-rh\tau} \leq V(x_k)\mathrm{e}^{-(1-\rho)h\tau} - V(x_k)\mathrm{e}^{-rh\tau} < 0,$$

where the last inequality follows from the fact that $r < (1-\rho)$. This implies that $\tau_2(x_k) \geq \tau_1(x_k)$ which completes the proof of claim(c). Now, by using Theorem 4 and claim (c), we can say that there exists a uniform positive lower bound on the inter-event times generated by the event-triggering rule (6) under the given conditions on the parameters r, ε and the model uncertainty $\|\tilde{A}_c\|$. □

Note that, in both Theorem 4 and Theorem 5, the condition on the model uncertainty is expressed through some bound on the modeling error $\tilde{A}_c$ that involves the error in both $\hat{A}$ and $\hat{B}$. Instead, we can also provide some conservative bounds on the individual errors in $\hat{A}$ and $\hat{B}$, respectively.

D. On Choosing the parameter T: Special Case of Polynomial Control Input

One of the important parameters in the proposed event-triggered parametrized control (ETPC) is the time horizon T on which the control input is fitted to approximate an ideal control input. Note that if the parameter ε, in the optimization problem (4) is bounded as in Theorems 5 and 4, $\dot{V}$ just after each event is guaranteed to be sufficiently less than zero. This property is independent of the value of T. Thus, T can take any arbitrary positive value and still ensure strictly positive inter-event times. Choosing very small value for T is likely to give some thing close to a first-order hold control since it would be akin to linearinzing the ideal control input at t_k. Choosing a very large T can lead to a greater fitting error in the short-term just after t_k and hence actually lead to smaller inter-event times. Thus, there is clearly a tradeoff for tuning the value of T. We cannot give insights into this problem for a general set of functions Φ. However, in the special case of Φ consisting of only polynomial functions, which is an important class of functions, we can provide some insights using Taylor's remainder theorem.

Specifically, consider the special case where the control input is a linear combination of the functions $\{1, \tau, \tau^2, \ldots, \tau^p\}$. In other words, the control input is a polynomial of degree p. Under this special case, we can give more insights into the effect of T, the length of the time horizon of the optimization problem (4), on the norm of the fitting error $u - K\hat{x}$.

Proposition 6. *(Effect of T on the performance of event-triggered polynomial control). Consider the system (1) under the parameterized control law (2), with $\Phi = \{1, \tau, \ldots, \tau^p\}$, whose coefficients are obtained by solving (4) for some $\varepsilon \geq 0$. Then, the norm of the polynomial fitting error $u - K\hat{x}$ has an upper bound which is an increasing function of T.*

Proof. For all $t \in [t_k, t_{k+1})$, $\exists j \in \{1, 2, \ldots, m\}$ such that,

$$\begin{aligned}\|u(t) - K\hat{x}(t)\| &\leq \sqrt{m}\|u(t) - K\hat{x}(t)\|_\infty = \sqrt{m}|u_j(t) - K_j\hat{x}(t)| \\ &\leq \sqrt{m}\int_0^T |u_j(t_k+\tau) - K_j e^{\hat{A}_c\tau}\hat{x}(t_k)|d\tau \\ &\leq \sqrt{m}\int_0^T |\bar{u}_j(t_k+\tau) - K_j e^{\hat{A}_c\tau}\hat{x}(t_k)|d\tau,\end{aligned}$$

where $\bar{u}_j(t_k+\tau) := K_j(I + \hat{A}_c\tau + \frac{\hat{A}_c^2\tau^2}{2!} + \ldots + \frac{\hat{A}_c^p\tau^p}{p!})\hat{x}(t_k)$. The last inequality follows from the fact that $\bar{u}_j(t_k+\tau)$ is a feasible solution of the optimization problem (4). Note that, $I + \hat{A}_c\tau + \ldots + \frac{\hat{A}_c^p\tau^p}{p!}$ is the p^{th} Taylor polynomial of $e^{\hat{A}_c\tau}$. By using Taylor's remainder theorem, we can say that $\left\|I + \hat{A}_c\tau + \ldots + \frac{\hat{A}_c^p\tau^p}{p!} - e^{\hat{A}_c\tau}\right\| \leq \frac{M}{(p+1)!}|\tau|^{p+1} \leq \frac{M}{(p+1)!}T^{p+1}$ for some $M > 0$. Thus,

$$\|u(t) - K\hat{x}(t)\| \leq \sqrt{m}\|K\|\,\|\hat{x}(t_k)\|\,\frac{M}{(p+1)!}T^{p+2},$$

which completes the proof of this proposition. □

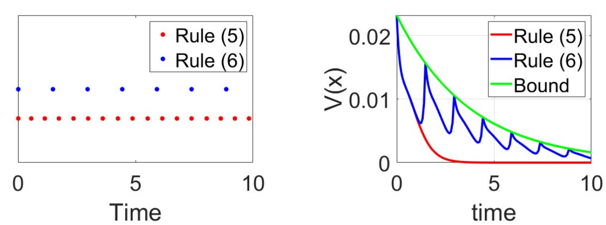
IV. NUMERICAL EXAMPLES

In this section, we present two numerical examples to illustrate our results.

Example 1: Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u =: Ax + Bu.$$

A has real eigenvalues at $\{1, 2\}$. Let the available model be $\hat{A} = A + 0.05I$ and $\hat{B} = \begin{bmatrix} 0 & 1.01 \end{bmatrix}^T$. The control gain $K = [-0.1510 \quad -6.0396]$ ensures that $\hat{A}_c := \hat{A} + \hat{B}K$ has real eigenvalues at $\{-1, -2\}$. Let $Q = \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 1.5 \end{bmatrix}$. We choose the parameters $\rho = 0.5$, $r = 0.45$, $\sigma = 0.1612$ and $\varepsilon = 0.031$. Here, all the parameters are chosen according to the bounds given in our results. The norm of the model uncertainty, $\tilde{A}_c := \hat{A}_c - (A + BK)$, also satisfies the given bound in our results. In this example, we consider the control input as a linear combination of the set of functions $\{1, \tau, \tau^2, \ldots, \tau^p\}$. Figure 2 presents the simulation results of example 1 with $p = 3$, $T = 1$ and $x(0) = [0.1 \quad 0.2]^T$. Figure 2a presents the triggering time instants for both the event-triggering rules (5) and (6). We can see that the number of triggering instants is less for rule (6) compared to rule (5). Figure 2b shows the convergence of $V(x)$ for both the event-triggering rules. As expected, $V(x)$ converges faster with rule (5) compared to that of rule (6).



(a) Triggering instants (b) Convergence of $V(x)$

Fig. 2: Simulation results of Example 1.

Next, we consider 100 initial conditions uniformly sampled from the unit sphere. Then, we calculate the average inter-event time (AIET) and minimum inter-event time (MIET) over 100 events for each initial condition. The average of the AIET and the minimum of the MIET over the set of initial conditions for different event-triggering rules with $T = 2$ and $p = 3$ are given in Table I. We repeat the procedure for different values of T and p, and the observations are tabulated in Table II and Table III, respectively. Table I shows that both the event-triggering rules (5) and (6) perform better in terms of the AIET and MIET compared to the event-triggered ZOH control, as commonly seen in the event-triggered control literature.

TABLE I: Average of AIET and minimum of MIET, over a set of initial conditions, for different event-triggered controllers in Example 1.

	Rule (5)	**Rule** (5) **with ZOH**	**Rule** (6)	**Rule** (6) **with ZOH**
Average of AIET	0.4853	0.0312	1.2212	0.3526
Minimum of MIET	0.0495	0.0070	1.1759	0.3046

Table II shows that as T, the length of the time horizon of the optimization problems (4), increases both the average of AIET and minimum of MIET decrease for both the event-triggering rules (5) and (6). Table III shows that as p, the degree of the polynomial control input, increases both the average of AIET and minimum of MIET also increase for both the event-triggering rules (5) and (6).

TABLE II: Average of AIET and minimum of MIET, over a set of initial conditions, for different values of T with $p=3$ in Example 1.

	Average of AIET		**Minimum of MIET**	
T	**Rule** (5)	**Rule** (6)	**Rule** (5)	**Rule** (6)
1.5	0.5666	2.1745	0.0903	1.8110
2	0.4853	1.2212	0.0495	1.1759
2.5	0.3932	0.8981	0.0341	0.8174

TABLE III: Average of AIET and minimum of MIET, over a set of initial conditions, for different values of p with $T=2$ in Example 1.

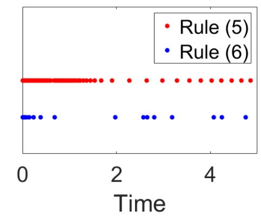
	Average of AIET		**Minimum of MIET**	
p	**Rule** (5)	**Rule** (6)	**Rule** (5)	**Rule** (6)
3	0.4853	1.2212	0.0495	1.1759
5	0.6187	1.7349	0.4263	1.6704
6	0.6935	1.7954	0.4846	1.6952

We also compare the performance of our control method with the model-based event-triggered control method [8]. The average of AIET and minimum of MIET, respectively, over a set of initial conditions for the model-based event-triggered control method with the triggering rule (5) are obtained as 0.6066 and 0.4585. For the same event-triggering rule, our method has higher values for both the average of AIET and minimum of MIET with $T=2$ and $p=6$.

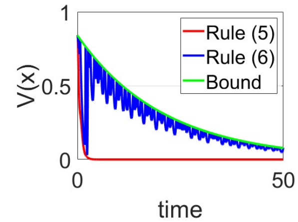
Example 2: Next, we consider a 5^{th} order system,

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 4 & -16 & 25 & -19 & 7 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u =: Ax + Bu.$$

A has real eigenvalues at $\{1,1,1,2,2\}$. Let the available model be $\hat{A} = A + 0.01I$ and $\hat{B} = B$. The control gain K is designed such that $\hat{A}_c$ has real eigenvalues at $\{-1,-1.5,-2,-2.5,-3\}$. Let $Q = I$ and we choose the parameters $\rho = 0.38$, $r = 0.6$, $\sigma = 0.17$ and $\varepsilon = 0.008$. Here also, all the parameters are chosen according to the bounds given in our results. The norm of the model uncertainty satisfies the given bound in our results. In this example also, we consider the control input as a polynomial of degree p.



(a) Triggering instants



(b) Convergence of $V(x)$

Fig. 3: Simulation results of Example 2.

Figure 3 presents the simulation results for example 2 with $p=3$, $T=1$ and $x(0) = [0.1 \quad 0.2 \quad 0.1 \quad 0.4 \quad 0.3]^T$. Figure 3a presents the triggering time instants for both the event-triggering rules (5) and (6). As in the previous example, the number of triggering instants is less for rule (6) compared to rule (5). Figure 3b shows the convergence of $V(x)$ for both the event-triggering rules. $V(x)$ converges much faster with rule (5) compared to that of rule (6).

As we did for the previous example, we calculate the average of AIET and minimum of MIET over a set of 100 initial conditions uniformly sampled from the unit sphere for different event-triggering rules with $T=0.5$, $p=3$ and are given in Table IV. We repeat the procedure for different values of T and the observations are tabulated in Table V. Table IV shows that both the event-triggering rules (5) and (6) perform significantly better in terms of the AIET and MIET compared to the event-triggered ZOH control.

TABLE IV: Average of AIET and minimum of MIET, over a set of initial conditions, for different event-triggering rules in Example 2.

	Rule (5)	**Rule** (5) **with ZOH**	**Rule** (6)	**Rule** (6) **with ZOH**
Average of AIET	0.1467	0.0002	0.8823	0.0982
Minimum of MIET	0.0014	0.00015	0.0460	0.0029

Table V shows that as T, the length of the time horizon of the optimization problems (4), increases both the average of AIET and minimum of MIET decrease for both the event-triggering rules (5) and (6).

TABLE V: Average of AIET and minimum of MIET, over a set of initial conditions, for different values of T with $p=3$ in Example 2.

	Average of AIET		**Minimum of MIET**	
T	**Rule** (5)	**Rule** (6)	**Rule** (5)	**Rule** (6)
0.5	0.1467	0.8823	0.0014	0.0460
1	0.0034	0.4705	0.0004	0.0062
1.5	0.002	0.3072	0.0002	0.0037

The average of AIET and minimum of MIET, respectively, over a set of initial conditions for the model-based event-triggered control method with the triggering rule (5) are obtained as 0.2279 and 0.0733. For the same event-triggering rule, our method has comparable values of 0.2052 and 0.0691, respectively, for the average of AIET and minimum of MIET with $T = 0.2$ and $p = 3$. In general, we can expect model based control to generate larger inter-event times compared to our approach with the same event-triggering rule as the control signal in our approach is an approximation of the model based control signal.

V. CONCLUSION

In this paper, we proposed a new control method called event-triggered parameterized control (ETPC) for stabilization of linear systems. In this control method, the control input to the plant is a linear combination of a set of linearly independent scalar functions that are continuously differentiable. At each event, the coefficients of the parameterized control input are chosen by minimizing the error in fitting to a model based control signal. The parameters of the updated control are then communicated to the actuator in an event-triggered manner. We designed two event-triggering rules that guarantee global asymptotic stability of the origin of the closed loop system. We also showed that both the event-triggering rules do not exhibit Zeno behavior for the generated inter-event times. We illustrated our results through numerical examples. We also compared the proposed control method with the existing event-triggered control based on zero-order-hold and showed a significant improvement in terms of the average inter-event time.

Some advantages of the proposed control method is better utilization of communication resources, lesser requirements on computational resources at the actuator compared to model based control and overcomes the synchronization, privacy and security issues present in model based control.

Future work includes the generalization of this control method to nonlinear and distributed control settings, analytical method to determine an optimal time horizon for function fitting, control under quantization of the parameters, time delays, desynchronized controller and actuator clocks, and a control Lyapunov function or MPC approach to ETPC.

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