# Event-Triggered Decentralized Dynamic Output Feedback Control for LTI Systems ⋆

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**Abstract:** In this paper we investigate event-triggered controllers for dynamic output feedback control of Linear Time Invariant (LTI) systems. We propose a systematic methodology for designing implicitly verified event-triggered dynamic output feedback controllers for LTI systems that are observable and controllable. Event-triggering conditions for sampled-data implementation of the observer and the controller are proposed for a decentralized architecture. The output of each sensor and actuator are asynchronously sampled by event-triggers that depend only on local information. The proposed controllers are shown to guarantee global asymptotic stability of the closed loop system, and provide global lower-bounds on inter-communication times. The proposed design methodology is illustrated through simulation results.

*Keywords:* Sampled data control, Linear output feedback, Decentralized control, Distributed control, Communication control applications, Linear control systems

## 1. INTRODUCTION

Recently, the event-triggered paradigm for sampling and control has been proposed as an alternative to the traditional time-triggered (example: periodic triggering) paradigm. In event based control systems, the timing of control execution is not necessarily periodic and is implicitly determined by a state dependent event-triggering condition. Thus, by encoding the nature of the task (for example stabilization) into an event-triggering condition, it is possible to design controllers that make better use of computational and communication resources.

In recent years, event-triggered controllers have been systematically designed (a representative list includes Tabuada (2007); Heemels et al. (2008); Wang and Lemmon (2010); Tallapragada and Chopra (2011)) for different stabilization tasks. Most work in this literature assumes the availability of full state information. However, in many practical applications only a part of the state information can be directly measured and a dynamic (for example, observer based) output feedback controller must be utilized.

The main **contribution** of this paper is a methodology for designing implicitly verified event-triggered dynamic output feedback controllers for Linear Time Invariant (LTI) systems that are observable and controllable. We consider a decentralized architecture, in which the output of each sensor is transmitted to a central controller asynchronously and each output of the controller are transmitted asynchronously. All the transmission times are determined by local event-triggers that depend only on local information and explicit positive lower thresholds for inter-sampling times. These thresholds are designed a priori by first expressing the dynamics of the measurement errors, resulting from sampling, in terms of the unknown state of the closed loop system and then finding a lower bound on the inter-sampling times that is independent of the unknown closed loop state. The resulting event-triggering conditions are shown to ensure global asymptotic stability of the closed loop system as well as global minimum inter-sampling times.

In the literature, among the few works that consider this problem, Donkers and Heemels (2010) proposed an event-triggered implementation that can guarantee uniform ultimate boundedness of the plant state and provided an estimate of minimum inter-communication time that holds semi-globally, whereas our proposed controller guarantees asymptotic stability and an estimate of inter-communication times that holds globally.

In Lehmann and Lunze (2011), a model based output feedback controller was proposed, where the communication from the observer subsystem to the system model subsystem is triggered by a condition that compares the observer state with that of a local copy of system model subsystem. Again, the controller guarantees only uniform ultimate boundedness of the closed loop state. In Li and Lemmon (2010, 2011) an output feedback control implementation for *discrete-time* systems is considered as an optimal control problem. The proposed architecture includes a Kalman filter in the sensor subsystem and identical observers in the sensor as well as actuator subsystems. The results provide an upper bound on the optimal cost attained by the event-triggered system. Another major

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difference of our proposed design with these works is that we do not require identical models/observers to be run at different locations.

Recently, in Almeida et al. (2012) (and references therein), a self-triggered dynamic output feedback controller was presented. The design involves a discrete time observer in cascade with a self-triggered controller designed for full state feedback, which is then shown to render the closed loop system input-to-state stable (ISS) with respect to exogenous disturbances. It would be interesting to see, in future, how these results can be utilized to design improved event-triggered output feedback controllers.

Among other works that consider event-triggered controllers for decentralized architectures, Mazo Jr. and Cao (2011); Wang and Lemmon (2011) (and references therein), it is assumed that each state of the system is measured by some sensor directly. To the best of our knowledge, asynchronous decentralized (in the sense described above) output feedback control is an open problem.

The rest of the paper is organized as follows. Section 2 describes the problem under consideration in this paper. In Section 3, the design of an event-triggered dynamic controller is presented - first for the centralized architecture, as a way of introduction and for comparison, and then for the decentralized architecture. The proposed design methodology is illustrated through simulations in Section 4 and finally Section 5 provides some concluding remarks.

## 2. PROBLEM STATEMENT

Consider a Multi Input Multi Output (MIMO) Linear Time Invariant (LTI) control system

$$\begin{aligned} &\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, \; u \in \mathbb{R}^m \\ &y = Cx, \quad y \in \mathbb{R}^p \end{aligned} \tag{1}$$

**Assume** that $(A, B)$ and $(A, C)$ are controllable and observable, respectively. Then, there exists a continuous-time dynamic controller (observer based controller)

$$\dot{\hat{x}} = (A + FC)\hat{x} + BK\hat{x} - Fy, \quad \hat{x} \in \mathbb{R}^n \tag{2}$$

$$u = K\hat{x} \tag{3}$$

such that the closed loop system (1)-(3) is globally asymptotically stable. More precisely, there exist gain matrices $F \in \mathbb{R}^n \times \mathbb{R}^p$ and $K \in \mathbb{R}^m \times \mathbb{R}^n$ such that $(A + FC)$ and $(A + BK)$ are Hurwitz. Let us denote the *observer estimation error* as

$$\tilde{x} \triangleq \hat{x} - x$$

Let $\psi \triangleq [x; \tilde{x}]$ be the aggregate state vector, where the notation $[a_1; a_2]$ denotes the column vector formed by concatenating the vectors $a_1$ and $a_2$. Then, the closed loop system may be written as

$$\dot{\psi} = \begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A + BK & BK \\ 0 & A + FC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} \triangleq \bar{A}\psi \tag{4}$$

where 0 represents a matrix of zeroes of appropriate size. There exist gains $F$ and $K$ such that the matrix $\bar{A}$ is Hurwitz, thus rendering the closed loop system stable.

In this paper, we are interested in event-triggered implementation of the dynamic controller (2)-(3). In sampled-data implementations (of which event-triggered implementation is an example) the data used by the controller and actuator are sampled at discrete time instants. In the traditional time-triggered implementation the signals are sampled at periodic intervals or at pre-determined time instants. On the other hand, in event-triggered implementation, the time instants at which the signals are sampled are determined implicitly by a state/data based triggering condition at run-time.

Before we proceed, it is useful to look at some important notational conventions adopted in this paper. Depending on the context, the notation $\|.\|$ denotes the Euclidean norm or the induced matrix Euclidean norm. Next, let $z$ be any continuous-time signal. Let $\{t_i^z\}$ be the increasing sequence of time instants at which the signal $z$ is sampled. Then we denote the sampled signal by $z_s$, that is,

$$z_s \triangleq z(t_i^z), \quad \forall t \in [t_i^z, t_{i+1}^z)$$

The sampled signal, $z_s$, is thus piece-wise constant in time. It is common in the event-triggered control literature to view the sampled data as resulting from an error in the continuous-time measurement of the signal $z$. This measurement error is denoted by

$$z_e \triangleq z_s - z = z(t_i^z) - z, \quad \forall t \in [t_i^z, t_{i+1}^z)$$

In time-triggered implementations, the time instants $t_i^z$ are pre-determined. However, in event-triggered implementations the time instants $t_i^z$ are determined implicitly by a triggering condition. Due to this, an event-triggering condition may result in the inter sampling times $t_{i+1}^z - t_i^z$ to be arbitrarily close to zero or it may even result in the limit of the sequence $\{t_i^z\}$ to be a finite number (*Zeno* behavior). Thus for practical utility, an event-trigger has to ensure that these scenarios do not occur.

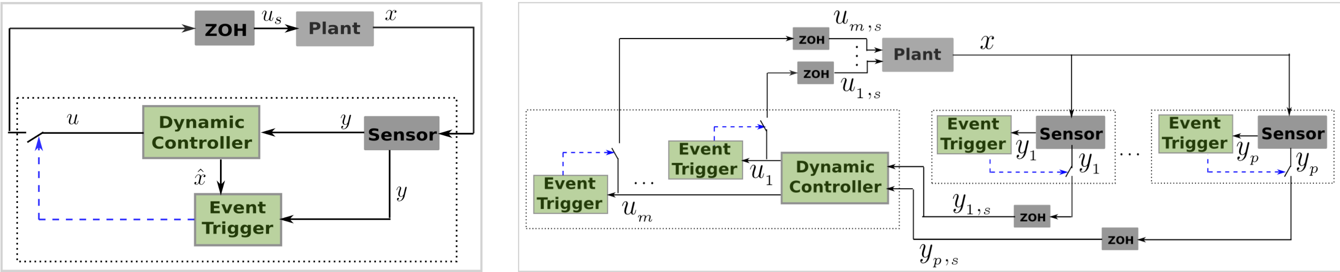
The design of an event-triggered dynamic output feedback controller depends critically on the architecture of the system. The simplest case is that of all the sensors and the controller being co-located, Figure 1a. *Co-located* components have access to each others' outputs at all times. Thus, in this case the event-triggering condition can depend on all the measured outputs. If however, the sensors and the controller are not co-located then for each sensor, and actuator an independent event-trigger is required, which depends only on locally available measurements. In this paper we consider a decentralized architecture as shown in Figure 1b, where $y_j$, $u_i$, $y_{j,s}$ and $u_{i,s}$ are the $j^{\text{th}}$ output, $i^{\text{th}}$ input to the plant and their sampled versions, respectively. Thus, in this case the sampling of the sensor and controller outputs occurs asynchronously.

The problem we are interested in this paper is the design of the dynamic controller and the event-triggers for the decentralized architecture shown in Figure 1b. The resulting event-triggered controllers must render the closed loop system globally asymptotically stable and ensure that the inter-sampling times of each signal is lower bounded by a positive constant.

In the next section, the dynamic controller and the event-triggers are developed.

## 3. EVENT-TRIGGERED IMPLEMENTATION OF THE DYNAMIC CONTROLLER

In this section, the dynamic controller and the event-triggers are developed, first for the centralized architecture




(a) Centralized architecture: Sensor and controller co-located.



(b) Decentralized architecture: Centralized controller with distributed sensors and actuators, each transmitting data in parallel and asynchronously.

Fig. 1. The centralized and decentralized architectures for event-triggered dynamic output feedback control. The components within a dotted box are at the same location.

and then for the decentralized architecture. But first we present a Lemma that is a modified version of Corollary IV.1 from Tabuada (2007), which is useful in the design of event-triggers in the sequel.

*Lemma 1.* Let $\dot{x} = Ax + \sum_{i=1}^{q} B_i u_{i,e}$ be any linear control system where $u_{i,e} \triangleq u_{i,s} - u_i$, $u_i = K_i x$ for $i \in \{1, \dots, q\}$, with $u_{i,s}$ being the piecewise constant signals obtained by asynchronously sampling $u_i$ respectively. Let $w_i \geq 0$ for $i \in \{1, \dots, q\}$ be any arbitrary constants and let $W = \sum_{i=1}^{q} \|B_i\| w_i$. Suppose the sampling instants are such that for all time $\|u_{i,e}\|/\|x\| \leq w_i$ for each $i \in \{1, \dots, q\}$. Then, the time required for $\|u_{i,e}\|/\|x\|$ to evolve from 0 to $w_i$ is lower bounded by $\tau_i$, where

$$\tau_i = \tau(w_i, \|A\| + W - \|B_i\| w_i, \|B_i\|, \|K_i\|)$$

The function $\tau$ is given by

$$\tau(w, a, b, k) = \{t \geq 0 : \phi(t, 0) = w\} \tag{5}$$

where $a$, $b$, $k$ are positive constants and $\phi(t, c)$ is the solution of

$$\dot{\phi} = \big(k + \phi\big)\big(a + b\phi\big), \quad \phi(0, c) = c$$

**Proof.** Letting $\nu_i \triangleq \|u_{i,e}\|/\|x\|$ and by direct calculation we see that for $i \in \{1, \dots, q\}$

$$\begin{aligned} \frac{\mathrm{d}\nu_i}{\mathrm{d}t} &= \frac{-(u_{i,e}^T u_{i,e})^{-1/2} u_{i,e}^T K_i \dot{x}}{\|x\|} - \frac{x^T \dot{x} \|u_{i,e}\|}{\|x\|^3} \\ &\leq \big(\|K_i\| + \nu_i\big) \frac{\|\dot{x}\|}{\|x\|} \end{aligned}$$

$$\frac{\mathrm{d}\nu_i}{\mathrm{d}t} \leq \big(\|K_i\| + \nu_i\big) \frac{\|Ax\| + \sum_{j=1}^{q} \|B_j\| \|u_{j,e}\|}{\|x\|}$$

where for $u_{i,e} = 0$ the relation holds for all directional derivatives. This relation is further simplified by enforcing the condition that the sampling instants are such that for all time $\nu_i \leq w_i$ for each $i \in \{1, \dots, q\}$.

$$\frac{\mathrm{d}\nu_i}{\mathrm{d}t} \leq \big(\|K_i\| + \nu_i\big)\big(\|A\| + W - \|B_i\| w_i + \|B_i\| \nu_i\big)$$

from which the claim of the Lemma directly follows. ∎

Note that in the above Lemma, $u_i$ can be scalars as well as vectors.

### *3.1 Centralized Architecture*

As a way of introduction, the event-triggered output feedback controller for the centralized architecture will be presented first. This is also useful in comparing the efficiency (in terms of the average control update frequency) of the decentralized architecture with the centralized architecture. Detailed results for the centralized architecture and other variations are presented in Tallapragada and Chopra (2012).

In the centralized architecture the observer and the sensor are co-located, which means the observer has access to the sensor information at all times. Now consider a sampled data version of the observer and the controller, that is

$$\dot{x} = Ax + Bu_s, \quad y = Cx \tag{6}$$

$$\dot{\hat{x}} = (A + FC)\hat{x} + BK\hat{x}_s - Fy, \quad u = K\hat{x} \tag{7}$$

where the subscript $s$ denotes the sampled versions of the corresponding continuous-time signals. The purpose of the term $BK\hat{x}_s$ in the observer (7) is to model the effect of the control $Bu_s = BK\hat{x}_s$ in the plant dynamics. Hence, (7) is the natural extension of the original observer (2).

The closed loop system may be written in terms of the measurement error $\hat{x}_e = \hat{x}_s - \hat{x}$ as

$$\begin{aligned} \dot{\psi} &= \bar{A}\psi + \begin{bmatrix} BK \\ 0 \end{bmatrix} \hat{x}_e = \bar{A}\psi + \begin{bmatrix} BK \\ 0 \end{bmatrix} [I_n \ \ I_n]\, \psi_e \\ &= \bar{A}\psi + G_1 H_1 \psi_e, \text{ where } G_1 \triangleq \begin{bmatrix} BK \\ 0 \end{bmatrix}, \ H_1 \triangleq [I_n \ \ I_n] \end{aligned} \tag{8}$$

where $\psi = [x; \tilde{x}] = [x; \hat{x} - x]$, $\psi_e = \psi_s - \psi$, $\bar{A}$ is as defined in (4), 0 represents a matrix of zeroes of appropriate dimension and $I_n$ is the $n \times n$ identity matrix. In this centralized architecture, $q = 1$ in the notation of Lemma 1 because all the controller outputs are updated synchronously. Also note that the sampled-data nature of the system is implicit in the measurement error term, $\hat{x}_e$ (or $\psi_e$). Hence, the system description is complete only with the specification of the event-triggering condition.

Now, the event-triggering condition is introduced in the following result, which also demonstrates that the resulting closed loop system is globally asymptotically stable and that the inter-sampling times are uniformly lower bounded by a positive constant.

*Theorem 1.* Let $Q \in \mathbb{R}^{2n}$ be any positive definite matrix. Consider the system given by (8) with $(A, B)$ controllable,

$(A, C)$ observable and $\bar{A}$ Hurwitz. Let the event-triggering condition be

$$t_{i+1} = \min\{t \geq t_i + T : \eta \geq 0\} \tag{9}$$

$$\eta = 2\|PG_1\|\|\hat{x}_e\| - \sigma Q_m \left( \frac{\theta_{\hat{x}}\|\hat{x}\|}{\sqrt{2}} + \frac{\theta_y\|y\|}{\|C\|} \right) \tag{10}$$

where $Q_m$ is the smallest eigenvalue of $Q$, $0 < \sigma < 1$, $0 \leq \theta_{\hat{x}} \leq 1$, $0 \leq \theta_y \leq 1$ are design parameters such that $\theta_{\hat{x}} + \theta_y = 1$ and $P > 0$ satisfies $P\bar{A} + \bar{A}^T P = -Q$. The inter-sampling time threshold is given by $T = \tau\left( \frac{\sigma Q_m}{2\|PG_1\|}, \|\bar{A}\|, \|G_1\|, \|H_1\| \right)$, the function $\tau$ being as defined in (5). Then, the origin of the closed loop system is globally asymptotically stable and the inter-sample times are lower bounded by $T$.

**Proof.** First note that $\bar{A}$ is Hurwitz and hence there exists a positive definite matrix $P$ that satisfies the Lyapunov condition in the statement of the theorem. Now consider the candidate Lyapunov function $V = \psi^T P \psi$, whose derivative along the flow of the closed loop system (8) is given by

$$\begin{aligned} \dot{V} &= \psi^T P \bar{A} \psi + \psi^T \bar{A}^T P \psi + 2\psi^T P G_1 H_1 \psi_e \\ &\leq -\psi^T Q \psi + 2\psi^T P G_1 \hat{x}_e \\ &\leq -(1-\sigma) Q_m \|\psi\|^2 - \|\psi\| \Big[ \sigma Q_m \|\psi\| - 2\|PG_1\|\|\hat{x}_e\| \Big] \end{aligned} \tag{11}$$

Now, if $\hat{x} \neq 0$ then there is a non-negative number $k$ such that $\|x\| = k\|\hat{x}\|$, from which it follows that

$$\begin{aligned} \|\psi\| &= \sqrt{\|x\|^2 + \|\hat{x} - x\|^2} \geq \sqrt{2\|x\|^2 + \|\hat{x}\|^2 - 2\|x\|\|\hat{x}\|} \\ &= \|\hat{x}\| \sqrt{2k^2 - 2k + 1} \geq \frac{\|\hat{x}\|}{\sqrt{2}} \end{aligned}$$

where the last step follows by minimizing with respect to $k$. If, on the other hand, $\hat{x} = 0$ then $\|\psi\| \geq \|\hat{x}\|/\sqrt{2}$ holds trivially. Next, we see that $\|\psi\| \geq \|x\| \geq \|y\|/\|C\|$. Thus

$$\|\psi\| \geq \frac{\theta_{\hat{x}}\|\hat{x}\|}{\sqrt{2}} + \frac{\theta_y\|y\|}{\|C\|}, \quad \text{with } \theta_{\hat{x}} + \theta_y = 1$$

Hence the triggering condition (9) ensures that

$$\dot{V} \leq -(1-\sigma) Q_m \|\psi\|^2$$

from which it follows that the origin of the closed loop system is globally asymptotically stable. Next, from (11), it is clear that $\dot{V} \leq -(1-\sigma) Q_m \|\psi\|^2$ as long as

$$\frac{\|\hat{x}_e\|}{\|\psi\|} \leq w = \frac{\sigma Q_m}{2\|PG_1\|}$$

Thus the inter-sample times are lower bounded by $T$, which is positive because $w > 0$ and the norms of all the system matrices are finite. ■

Note that if $(A, B)$ is controllable and $(A, C)$ is observable, then there exist gains $K$ and $F$ such that $\bar{A}$ is Hurwitz. Next, the time for the function $\eta$, (10), to grow to zero from its initial value at a sampling instant can be arbitrarily small. However, a lower bound on the time for which $\dot{V}$ remains negative after sampling was found from (11) and Lemma 1. This key step is possible because $\hat{x}_e$ is a linear function of $\psi_e$ and Lemma 1 provides a lower bound for inter-sampling times, which is independent of the unknown state of the closed loop system $\psi$.

In the next subsection, we design the dynamic controller and the event-triggers for the decentralized architecture of Figure 1b, where the sensors are distributed and the each controller output is updated in parallel and asynchronously.

### *3.2 Decentralized Architecture*

In the decentralized architecture of Figure 1b, the sensors are distributed. Their outputs are sampled and communicated to a central controller asynchronously by independent event-triggers that depend only on local information. Further, the different controller outputs are updated in parallel and asynchronously. The closed loop system is given by

$$\begin{aligned} \dot{x} &= Ax + Bu_s, \quad y = Cx \\ \dot{\hat{x}} &= (A + FC)\hat{x} + Bu_s - Fy_s, \quad u = K\hat{x} \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state of the plant, $\hat{x} \in \mathbb{R}^n$ is the observer state, $y \in \mathbb{R}^p$ is the vector of sensor outputs, $u \in \mathbb{R}^m$ is the vector of inputs to the plant from the actuators. The subscript $s$ denotes the sampled versions of the corresponding continuous-time signals.

It is to be understood implicitly that each actuator output $u_{i,s}$, for $i \in \{1, \ldots, m\}$, and each sensor output $y_{j,s}$, for $j \in \{1, \ldots, p\}$, represent asynchronously sampled signals, that is,

$$\begin{aligned} u_{i,s} &= u(t_k^{u_i}), \quad \forall t \in [t_k^{u_i}, t_{k+1}^{u_i}) \\ y_{j,s} &= y(t_k^{y_j}), \quad \forall t \in [t_k^{y_j}, t_{k+1}^{y_j}) \end{aligned}$$

It is possible to define $u_i$ and $y_j$ as vectors with only minor changes in notation. However, in this paper we restrict to the scalar case for simplicity.

Let us now denote the $i^{\text{th}}$ column of $B$ and $F$ by $B_i$ and $F_i$; and similarly, the $i^{\text{th}}$ row of $K$ and $C$ by $K_i$ and $C_i$, respectively. That is,

$$B \triangleq [B_1 \ \ldots \ B_m], \quad F \triangleq [F_1 \ \ldots \ F_p]$$

$$K \triangleq \begin{bmatrix} K_1 \\ \vdots \\ K_m \end{bmatrix}, \quad C \triangleq \begin{bmatrix} C_1 \\ \vdots \\ C_p \end{bmatrix}$$

$$\tilde{B}_i \triangleq \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ \tilde{F}_j \triangleq \begin{bmatrix} 0 \\ F_j \end{bmatrix}, \ \tilde{K}_i \triangleq K_i [I_n \ I_n], \ \tilde{C}_j \triangleq [C_j \ 0]$$

where 0 represents a vector of the same size as $B_i$, $F_j$ and $C_j$ in $\tilde{B}_i$, $\tilde{F}_j$ and $\tilde{C}_j$, respectively. The notation $I_n$ denotes the $n \times n$ identity matrix.

Let $\psi \triangleq [x; \tilde{x}] \triangleq [x; \hat{x} - x]$ be the aggregate state vector, where the notation $[a_1; a_2]$ denotes the column vector formed by concatenating the vectors $a_1$ and $a_2$. Then, the closed loop system may be written in terms of the sampling/measurement errors, $u_{i,e} = u_{i,s} - u_i$ and $y_{j,e} = y_{j,s} - y_j$, as

$$\dot{\psi} = \bar{A}\psi + \sum_{i=1}^{m} \tilde{B}_i u_{i,e} - \sum_{j=1}^{p} \tilde{F}_j y_{j,e} \tag{12}$$

where $\bar{A}$ is as defined in (4). Thus for this architecture, $q = p + m$ in the notation of Lemma 1. Note that the sampled-data nature of the system is implicit in the measurement error terms, $u_{i,e}$ and $y_{j,e}$. Also note that the sensor outputs and the controller outputs can be expressed in terms of the full closed loop state as

$$u_i = K_i \hat{x} = \tilde{K}_i \psi, \qquad y_j = C_j x = \tilde{C}_j \psi \tag{13}$$

Now, the event-triggering condition is introduced in the following result, which also demonstrates that the resulting closed loop system is globally asymptotically stable and that the inter-sampling times are uniformly lower bounded by a positive constant.

*Theorem 2.* Let $Q \in \mathbb{R}^{2n}$ be any positive definite matrix. Consider the closed loop system given by (12) with $(A, B)$ controllable, $(A, C)$ observable and $\bar{A}$ Hurwitz. Let the event-triggering conditions be

$$t_{k+1}^{u_i} = \min\{t \geq t_k^{u_i} + T_{u,i} : \eta_{u_i} \geq 0\}, \forall i \in \{1, \ldots, m\} \tag{14}$$

$$t_{k+1}^{y_j} = \min\{t \geq t_k^{y_j} + T_{y,j} : \eta_{y_j} \geq 0\}, \forall j \in \{1, \ldots, p\} \tag{15}$$

$$\eta_{u_i} = 2\|\tilde{K}_i\|\|P\tilde{B}_i\|\|u_{i,e}\| - \sigma Q_m \theta_{u,i}\|u_i\| \tag{16}$$

$$\eta_{y_j} = 2\|\tilde{C}_j\|\|P\tilde{F}_j\|\|y_{j,e}\| - \sigma Q_m \theta_{y,j}\|y_j\| \tag{17}$$

where $Q_m$ is the smallest eigenvalue of $Q$, $0 < \sigma < 1$, $0 < \theta_{u,i} < 1$, $0 < \theta_{y,j} < 1$ are design parameters such that $\sum \theta_{u,i} + \sum \theta_{y,j} = 1$ and $P > 0$ satisfies $P\bar{A} + \bar{A}^T P = -Q$. The inter-sampling time thresholds are given by

$$T_{u,i} = \tau(w_{u,i}, \|\bar{A}\| + W - \|\tilde{B}_i\| w_{u,i}, \|\tilde{B}_i\|, \|\tilde{K}_i\|)$$

$$T_{y,j} = \tau(w_{y,j}, \|\bar{A}\| + W - \|\tilde{F}_j\| w_{y,j}, \|\tilde{F}_j\|, \|\tilde{C}_j\|)$$

where $W = \sum \|\tilde{B}_i\| w_{u,i} + \sum \|\tilde{F}_j\| w_{y,j}$, the function $\tau$ is defined as in (5) while $w_{u,i} = \dfrac{\sigma Q_m \theta_{u,i}}{2\|P\tilde{B}_i\|}$, $w_{y,j} = \dfrac{\sigma Q_m \theta_{y,j}}{2\|P\tilde{F}_j\|}$. Then, the origin of the closed loop system is globally asymptotically stable and the inter-sample times of $u_i$ and $y_j$ are lower bounded by $T_{u,i}$ and $T_{y,j}$, respectively.

**Proof.** Consider the candidate Lyapunov function $V = \psi^T P \psi$, whose derivative along the flow of the closed loop system is given by

$$\dot{V} = \psi^T(P\bar{A}\psi + \bar{A}^T P)\psi + 2\psi^T P\left[\sum_{i=1}^{m} \tilde{B}_i u_{i,e} - \sum_{j=1}^{p} \tilde{F}_j y_{j,e}\right]$$

$$\leq -\psi^T Q \psi + 2\psi^T P\left[\sum_{i=1}^{m} \tilde{B}_i u_{i,e} - \sum_{j=1}^{p} \tilde{F}_j y_{j,e}\right] \tag{18}$$

Notice that

$$\|u_i\| \leq \|\tilde{K}_i\|\|\psi\|, \quad \|y_j\| \leq \|\tilde{C}_j\|\|\psi\|$$

Thus it is easy to see that

$$\dot{V} \leq -(1-\sigma)Q_m\|\psi\|^2 + \|\psi\|\left[\sum_{i=1}^{m} \frac{\eta_{u_i}}{\|\tilde{K}_i\|} + \sum_{j=1}^{p} \frac{\eta_{y_j}}{\|\tilde{C}_j\|}\right]$$

The triggering conditions ensure that $\eta_{u_i} \leq 0$ and $\eta_{y_j} \leq 0$. Thus,

$$\dot{V} \leq -(1-\sigma)Q_m\|\psi\|^2 \tag{19}$$

from which it follows that the origin of the closed loop system is globally asymptotically stable. Moreover, it is clear from (18) that (19) holds as long as

$$\frac{\|u_{i,e}\|}{\|\psi\|} \leq w_{u,i} = \frac{\sigma Q_m \theta_{u,i}}{2\|P\tilde{B}_i\|}, \quad \frac{\|y_{j,e}\|}{\|\psi\|} \leq w_{y,j} = \frac{\sigma Q_m \theta_{y,j}}{2\|P\tilde{F}_j\|}$$

Note that each $w_{u,i}$ and $w_{y,j}$ are strictly positive while the norms of all the system matrices are finite. Hence, we conclude from Lemma 1 that the lower bounds for inter-sampling times of $u_i$ and $y_j$, $T_{u,i}$ and $T_{y,j}$, respectively are strictly positive. ■

Note that in the proposed event-triggers a minimum sampling time is enforced by design. This is necessary because each event-trigger at the sensors and the controllers depends only on local information and not on the complete state of the system. Thus, each sensor and controller subsystem has external inputs, which means no lower bound can be guaranteed for zero crossings of $\eta_{u_i}$ and $\eta_{y_j}$. However, these external inputs are linear functions of the overall system state and hence bounds on the rates of evolution of $\frac{\|u_{i,e}\|}{\|\psi\|}$ and $\frac{\|y_{j,e}\|}{\|\psi\|}$ can be found independent of the unknown system state, $\psi$. From these bounds a minimum sampling time for each of the sensors and controllers can be found that ensure $\dot{V} < 0$. This is exactly what the procedure in Lemma 1 accomplishes. Finally, the proposed event-triggers only ensure that the inter-sample times of $u_i$ and $y_j$ are individually lower bounded by positive constants. It must be noted that it is possible for different controller outputs and sensor outputs to be sampled arbitrarily close to each other.

In the next section, simulation results are presented to illustrate the proposed event-triggered controllers.

## 4. SIMULATION RESULTS

In this section, the proposed event-triggered dynamic output feedback controllers are illustrated for a linearized model of a batch reactor, Walsh and Ye (2001). The plant and the dynamic controller are given by (1)-(3) with

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & -0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

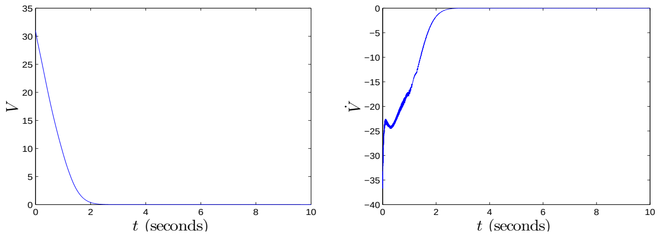
$$B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; F = -\begin{bmatrix} -2 & 0 \\ -4 & -1 \\ -2 & 2 \\ -1 & -4 \end{bmatrix}$$

$$K = -\begin{bmatrix} 0.1769 & 0.079 & 0.0794 & -0.2465 \\ 1.0328 & 0.1896 & -0.4479 & 0.7176 \end{bmatrix}$$

In the event-triggered controllers, $Q = I_8$, the $8 \times 8$ identity matrix, $\sigma = 0.95$ were chosen. For the simulations presented here, the initial condition of the plant and the observer were chosen as $x(0) = [2; 3; -1; 2]$ and $\hat{x}(0) = [0; 0; 0; 0]$, respectively. The zeroth sampling instant was chosen as $t_0^z = -T_z$ for each signal $z$. This is to ensure sampling at $t = 0$ if the $\psi_s(0)$ satisfies the triggering condition. In the centralized architecture, the initial sampled data was the same as the actual data. While in the decentralized architecture, the initial sampled data $u_s(0)$ and $y_s(0)$ were chosen arbitrarily to be consistent with the asynchronous sampling model. In the presented simulations $y_s(0) = [2; 3]$ and $u_s(0) = [1; 2]$ were chosen and finally the simulation time was chosen as 10s.

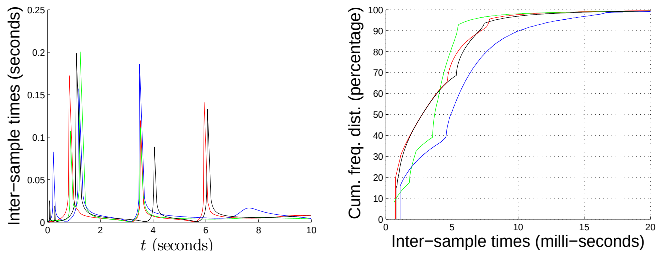
In the centralized architecture, $\theta_u = \theta_y = 0.5$ was chosen and the average inter-sample time was obtained as 0.0125s. In the decentralized architecture, the $\theta$ parameters were chosen as $\theta_u = [0.1\ 0.25]$ and $\theta_y = [0.35\ 0.3]$. The average inter-sample times obtained in this case were much higher at $[\bar{T}_{y,1}, \bar{T}_{y,2}, \bar{T}_{u,1}, \bar{T}_{u,2}] = 10^{-3} \times [3.6, 3.8, 5.6, 3.8]$s.

Figure 2 shows the evolution of the Lyapunov function and its derivative along the flow of the closed loop system. Figure 3 shows the inter-sample times and the cumulative frequency distribution of the inter-sample times in the decentralized architecture. The slower the rise of the cumulative distribution curves, the more useful the event-triggered controller is as opposed to a time-triggered one.




(a) (b)

Fig. 2. (a) The evolution of the Lyapunov function and (b) its derivative along the flow of the closed loop system in the decentralized architecture.




(a) (b)

Fig. 3. (a) Inter-sample times and (b) the cumulative frequency distribution of the inter-sample times in the decentralized architecture. The curves with colors red, green, blue and black correspond to sampling time data of $y_1$, $y_2$, $u_1$ and $u_2$, respectively.

Finally simulations were also performed for a decentralized time-triggered architecture with the constant sampling periods chosen as the observed average inter-sample times in the event-triggered case, that is $[\bar{T}_{y,1}, \bar{T}_{y,2}, \bar{T}_{u,1}, \bar{T}_{u,2}] = 10^{-3} \times [3.6, 3.8, 5.6, 3.8]$s. In this case too, it was observed that the state of the closed loop system asymptotically converged to zero, with the derivative of the Lyapunov function along the flow of the system always being negative. Similar results were obtained for several other initial conditions. Thus, the parameters $T_{u,i}$ and $T_{y,j}$ in the triggering conditions are very conservative and can be improved.

## 5. CONCLUSIONS

In this paper, an event-triggered dynamic output feedback controller design has been developed for a decentralized architecture. The designed event-triggering conditions have been shown to ensure global asymptotic stability of the origin of the closed loop system. A positive lower bound on inter-sampling times of the controller and the sensor outputs is guaranteed by explicitly including a lower threshold on inter-sampling interval in the event-triggering conditions. The design of these thresholds was also presented. The proposed controllers were illustrated through simulations.

The minimum inter-sample time thresholds in the triggering conditions were found to be very conservative. Thus, in future, these estimates have to be improved significantly and alternative event-triggering conditions have to be investigated. The gains and other parameters in the system were chosen in an ad hoc manner and hence systematic design methods are required for efficient controller design.

## REFERENCES


Almeida, J., Silvestre, C., and Pascoal, A.M. (2012). Observer based self-triggered control of linear plants with unknown disturbances. In *American Control Conference*, 5688–5693.

Donkers, M. and Heemels, W. (2010). Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved event-triggering. In *IEEE Conference on Decision and Control*, 3246–3251.

Heemels, W., Sandee, J., and Van Den Bosch, P. (2008). Analysis of event-driven controllers for linear systems. *International Journal of Control*, 81(4), 571–590.

Lehmann, D. and Lunze, J. (2011). Event-based output-feedback control. In *Mediterranean Conference on Control & Automation*, 982–987.

Li, L. and Lemmon, M. (2010). Event-triggered output feedback control of finite horizon discrete-time multi-dimensional linear processes. In *IEEE Conference on Decision and Control*, 3221–3226.

Li, L. and Lemmon, M. (2011). Weakly coupled event triggered output feedback control in wireless networked control systems. In *Annual Allerton Conference on Communication, Control, and Computing*, 572–579.

Mazo Jr., M. and Cao, M. (2011). Decentralized event-triggered control with asynchronous updates. In *IEEE Conference on Decision and Control and European Control Conference*, 2547 –2552.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9), 1680–1685.

Tallapragada, P. and Chopra, N. (2011). On event triggered trajectory tracking for control affine nonlinear systems. In *IEEE Conference on Decision and Control and European Control Conference*, 5377–5382.

Tallapragada, P. and Chopra, N. (2012). Event-triggered dynamic output feedback control for LTI systems. In *IEEE Conference on Decision and Control*. Submitted.

Walsh, G. and Ye, H. (2001). Scheduling of networked control systems. *IEEE Control Systems Magazine*, 21(1), 57–65.

Wang, X. and Lemmon, M. (2010). Self-triggering under state-independent disturbances. *IEEE Transactions on Automatic Control*, 55(6), 1494–1500.

Wang, X. and Lemmon, M. (2011). Event triggering in distributed networked control systems. *IEEE Transactions on Automatic Control*, 56(3), 586–601.