Event-Triggered Dynamic Output Feedback Control of LTI Systems over Sensor-Controller-Actuator Networks

Pavankumar Tallapragada and Nikhil Chopra

***Abstract*— This paper presents the design of event-triggered output feedback controllers for LTI systems over Sensor-Controller-Actuator Networks (SCAN). The SCAN is divided into three functional layers - sensor layer, controller layer and the actuator layer, each layer consisting of several nodes. The communication between the nodes is intermittent and event-triggered. Further, the flow of information occurs from the sensor to controller to actuator layer with the intra-layer communication occurring only in the controller layer. The event-triggers are designed to utilize only locally available information, making the nodes' transmissions asynchronous. The proposed control design guarantees global asymptotic stability of the origin of the system and a positive lower bound for the inter-transmission times of each node individually. The proposed design method is illustrated through simulation results.**

I. Introduction

Sensor-Controller-Actuator Networks (SCAN) consist of physically distributed nodes, each of which performs one or more of sensing, control computation and actuation tasks in order to control a plant. If the aggregate feedback provided by the sensor nodes does not constitute full state feedback, then the controller nodes may also have to distributively estimate the state of the plant. Interest in such networked control systems has been rising steadily - specially in the context of large scale systems such as power grids, building HVAC and even in vehicles. Some of the challenges in SCAN are asynchronous transmission of data; asynchronous and distributed computation; decision making based only on local information and time delays. Many of these features can be thought of as a manifestation of asynchronously sampled data. Further, in SCAN there are constraints on data rate, resources and energy. Given these factors, state based aperiodic event-triggering techniques have great potential for analyzing and designing SCAN.

In event based control systems, a state or data dependent event-triggering condition implicitly determines the (generally aperiodic) time instances at which control is updated or when a sensor transmits data to a controller. However, much of the literature on event-triggered control assumes the availability of full state information in the event-triggers, which is usually not possible in SCAN because no single node has complete state information or state estimate.

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In order for the proposed event-triggering technique to be applicable for a variety of architectures, we take a *functional* approach. We group the nodes in a SCAN into three *functional* layers - sensor layer, controller/observer layer and the actuator layer, with no two nodes being co-located. In practice though, several nodes from the same or different layers may be co-located. Any such scenario can simply be treated as a special case of the proposed framework. Further, the sensor nodes intermittently broadcast their data to the nodes in the observer (dynamic controller) layer. The nodes in the observer layer compute the state of the observer in a distributed manner, with each node in the observer layer intermittently broadcasting its data to other nodes in the same layer. Each of the actuator nodes also intermittently receives data from a corresponding unique observer node.

The **contribution** of this paper is a methodology for designing implicitly verified SCAN with event-triggered communication for dynamic output feedback control of Linear Time Invariant (LTI) systems. The proposed design methodology prescribes event-triggers that determine when each node transmits data. The event-triggers are designed to utilize only locally available information, making the nodes' transmissions asynchronous. The proposed design renders the equilibrium of the system, at the origin, globally asymptotically stable and guarantees a positive lower bound for the inter-transmission times of each node individually. This paper may be seen as an important extension of our previous work [1], [2] on event-triggered dynamic output feedback control to the case where along with distributed sensors and actuators, the dynamic controller is also implemented in a distributed manner by non-co-located nodes, which communicate with each other intermittently (based on event-triggering) and asynchronously.

Full state feedback distributed event-triggered control was studied in [3]–[6]. In [3], [4] the subsystems are assumed to be weakly coupled, which allowed the design of event-triggers depending on only local information. Our proposed design method requires much less restrictive assumptions. In [5], [6] asynchronous transmission of data by sensors to a central controller is triggered by local event-triggers. However, this design guarantees only semi-global practical stability (even for linear systems) if the sensors do not listen to the central controller. Compared to this work, our proposed design holds for Linear Time Invariant (LTI) systems with dynamic output feedback control and guarantees global asymptotic stability without the sensors having to listen to the distributed nodes of the controller/observer layer. In [7], asynchronous event-triggered dynamic output feedback control was studied, though the proposed method utilizes a centralized dynamic controller and guarantees only semi-global practical stability. Recently, [8] proposed a method for designing continuous time distributed observers with discrete communication. In this paper, the sensor and the observer for the i^{th} subsystem are co-located and additionally, an observability condition for each of the individual subsystems was assumed. Compared to [8], the current paper considers non-co-located sensor and observer nodes, requires an observability condition only for the overall system and further, distributed dynamic control is also implemented.

II. Problem Setup

Consider a Multi Input Multi Output (MIMO) Linear Time Invariant (LTI) control system

$$\dot{x} = Ax + Bu, \quad y = Cx \tag{1}$$

$$\dot{\hat{x}} = (A + FC)\hat{x} + Bu - Fy, \quad u = K\hat{x} \tag{2}$$

where $x \in \mathbb{R}^n$ is the plant state, $\hat{x} \in \mathbb{R}^n$ is the observer state, $y \in \mathbb{R}^p$ is the output of the plant and $u \in \mathbb{R}^m$ are the m actuator inputs to the plant. The matrices A, B, C, F and K are of appropriate dimensions. Denoting the *observer estimation error* and the state of the closed loop system, respectively, as

$$\tilde{x} \triangleq \hat{x} - x, \quad \psi \triangleq [x^T, \tilde{x}^T]^T$$

where the notation $[x^T, \tilde{x}^T]^T$ denotes the vector formed by concatenating the column vectors x and $\tilde{x}$, the closed loop system may be written as

$$\dot{\psi} = \begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A + BK & BK \\ \mathbf{0}_{n,n} & A + FC \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} \triangleq \bar{A}\psi \tag{3}$$

where $\mathbf{0}_{n,n}$ represents the $n \times n$ matrix of zeroes. The origin of the closed loop system (3) is globally asymptotically stable if and only if the matrix $\bar{A}$ is Hurwitz. Typically, (A, B) and (A, C) are assumed to be controllable and observable, respectively. This is sufficient to design the gain matrices F and K (which exist) such that $(A + FC)$, $(A + BK)$ and hence $\bar{A}$ are Hurwitz. For our purpose here, it is sufficient to assume $\bar{A}$ is Hurwitz.

In this paper, we are interested in a decentralized implementation of the dynamic controller (2) with event-triggered communication. In sampled-data implementations (of which event-triggered implementation is an example) the controller and or the actuator use sampled versions of signals. Formally, let ζ be any continuous-time signal (scalar or vector) and let $\{t_i^\zeta\}$ be the increasing sequence of time instants at which ζ is sampled. Then we denote the resulting piecewise constant sampled data signal by ζ_s, that is,

$$\zeta_s \triangleq \zeta(t_i^\zeta), \quad \forall t \in [t_i^\zeta, t_{i+1}^\zeta) \tag{4}$$

and the "measurement error" due to sampling as

$$\zeta_e \triangleq \zeta_s - \zeta = \zeta(t_i^\zeta) - \zeta, \quad \forall t \in [t_i^\zeta, t_{i+1}^\zeta)$$

It is sometimes convenient (and intuitive) to group together asynchronously transmitted signals into a single vector. Therefore given a vector $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_{n_\zeta}]^T \in \mathbb{R}^{n_\zeta}$, let

$$\zeta_s^* \triangleq [\zeta_{1,s}, \zeta_{2,s}, \ldots, \zeta_{n_\zeta,s}]^T \tag{5}$$

where $\zeta_{k,s}$, for $k \in \{1, \ldots, n_\zeta\}$ are piecewise constant sampled data signals defined as in (4). The measurement error is correspondingly defined as

$$\zeta_e^* \triangleq \zeta_s^* - \zeta$$

In time-triggered implementations, the time instants t_i^ζ are pre-determined and are commonly a multiple of a fixed sampling period. On the other hand, in event-triggered implementations the time instants t_i^ζ are determined implicitly by a state/data based triggering condition that is checked online. Consequently, an event-triggering condition may cause the inter sampling times $t_{i+1}^\zeta - t_i^\zeta$ to be arbitrarily close to zero or it may even result in the limit of the sequence $\{t_i^\zeta\}$ to be a finite number (*Zeno* behavior). Thus for practical utility, an event-trigger has to ensure that these scenarios do not occur.

Figure 1 shows the control architecture under consideration in this paper. The control system contains three *functional* layers - the sensor layer, the dynamic controller/observer layer and the actuator layer. Each layer consists of non-co-located (distributed) nodes. The sensor, observer and the actuator layers consist of p, n and m nodes, respectively. In the figure, the solid arrows indicate physical links, while the dotted arrows indicate the links on which the communication is event-triggered. The event-trigger for each of these latter links is located at the tail end of the arrow and uses only information locally available at that node. Meanwhile, the node or the nodes at the receiving end utilize the asynchronously transmitted data (sampled data), indicated by the subscript s. Note that the arrows that go from an arbitrary node 'A' to a layer circle in the figure indicate broadcast communication from the node 'A' to all the nodes in the layer circle. The aggregate observer state $z = [z_1, \ldots, z_n]^T$ is simply a basis transformation of the vector $\hat{x}$ of (2). When this basis transformation is appropriately chosen, the communication from the observer layer to the actuator layer is simplified and the actuator inputs to the plant are $u_i = z_{i,s}$ for $i \in \{1, \ldots, m\}$.

Figure 1 is a functional description of the control system and also represents the most general case, where no two nodes are co-located. If some nodes are co-located, then each collection of co-located nodes need not utilize the sampled versions of the data. Of particular interest is the case where the observer node z_i is co-located with the actuator node u_i for $i \in \{1, \ldots, m\}$. This special case is briefly discussed in the sequel. Next, in order to keep the notation simple, the data at each node is assumed to be scalar. Our results can be generalized to the vector case with only minor changes in the notation. Finally, note that in this paper, the terms 'transmit', 'update' and 'sample' are used interchangeably.

The design requirements in this paper are: (i) global asymptotic stability of the closed loop system and (ii) a

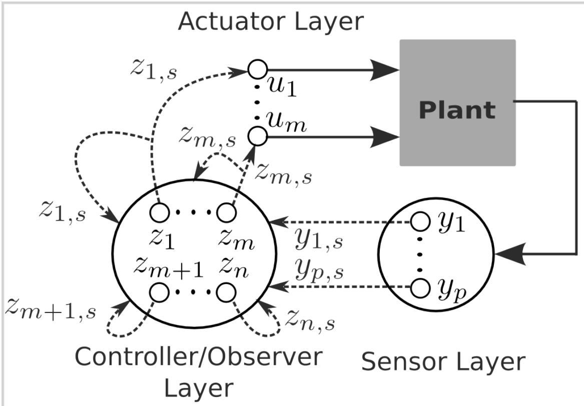


Fig. 1: The SCAN control system has three functional layers. Each node in the sensor layer intermittently broadcasts its output to all the nodes in the observer layer. Each node in the observer layer intermittently broadcasts its state to every other node in that layer. Each of the first m nodes of the observer layer also transmit intermittently to one of the actuator nodes. The dotted arrows indicate even-triggered communication links, with the event-trigger running at the tail end of the arrow. The solid arrows are physical links.

global positive lower bound for the inter-transmission times of each node. The proposed design procedure can be divided into two major stages. In the first stage, event-triggers are designed for asynchronous transmissions using centralized information. In the second stage, realizable event-triggers that depend only on local information are derived by appropriately under-approximating the centralized asynchronous event-triggers. The next section details this procedure in a general setting and in the subsequent section, it is applied to the problem at hand.

Note: In this paper, the notation $|.|$ is used to represent the Euclidean norm of a vector and also the induced Euclidean norm of a matrix.

III. Design of Decentralized Asynchronous Event-Triggering

This section presents the design of decentralized asynchronous event-triggering in a general setting so as to be applicable for multiple system architectures. Therefore, consider the system

$$\dot{\xi} = \mathcal{A}\xi + \sum_{j=1}^{q} \mathcal{B}_j \zeta_{j,s} = \mathcal{A}\xi + \mathcal{B}\zeta_s^* \tag{6}$$

where $\xi \in \mathbb{R}^{n_\xi}$, $\mathcal{A} \in \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_\xi}$, $\zeta_{j,s} \in \mathbb{R}^{d_j}$ is the sampled-data version of ζ_j, $\mathcal{B}_j \in \mathbb{R}^{n_\xi} \times \mathbb{R}^{d_j}$ is the j^{th} input matrix, $\zeta_s^* = [\zeta_{1,s}^T, \ldots, \zeta_{q,s}^T]^T \in \mathbb{R}^d$ is the asynchronously sampled-data version of ζ and is defined according to (5), $\mathcal{B} = [\mathcal{B}_1, \ldots, \mathcal{B}_q] \in \mathbb{R}^{n_\xi} \times \mathbb{R}^d$. Given a continuous-time feedback control law as

$$\zeta = \mathcal{K}\xi, \quad \zeta_j = \mathcal{K}_j\xi, \quad j \in \{1, \ldots, q\} \tag{7}$$

with $\mathcal{K}_j \in \mathbb{R}^{d_j} \times \mathbb{R}^{n_\xi}$ being the j^{th} block-row matrix of $\mathcal{K}$, the closed loop system with the sampled-data controller can be expressed as

$$\dot{\xi} = (\mathcal{A} + \mathcal{B}\mathcal{K})\xi + \mathcal{B}\zeta_e^* = \bar{\mathcal{A}}\xi + \mathcal{B}\zeta_e^* \tag{8}$$

where $\bar{\mathcal{A}} = (\mathcal{A} + \mathcal{B}\mathcal{K})$ and $\zeta_e^* = (\zeta_s^* - \zeta) \in \mathbb{R}^d$ is the measurement error due to sampling. Finally, suppose that the continuous time control law would have stabilized the closed loop system, that is,

(A1) Suppose that the matrix $\bar{\mathcal{A}}$ is Hurwitz, which ensures that for each symmetric positive definite matrix Q, there exists a symmetric positive definite matrix P such that $P\bar{\mathcal{A}} + \bar{\mathcal{A}}^T P = -Q$.

In order to develop the decentralized asynchronous event-triggers that implicitly specify the sampling time instants, $\{t_i^{\zeta_j}\}$, let us first consider the following result.

Lemma 1: Consider the sampled-data system (6) and assume (A1) holds. Let Q be any symmetric positive definite matrix and Q_m its smallest eigenvalue. For each $j \in \{1, \ldots, q\}$, let $\theta_j \in (0, 1)$, such that $\theta = \sum_{j=1}^{q} \theta_j \leq 1$ and

$$w_j = \frac{\sigma\theta_j Q_m}{|2P\mathcal{B}_j|} \tag{9}$$

where $\sigma \in (0, 1)$ is a design parameter. Suppose that for each $j \in \{1, \ldots, q\}$, the sampling instants $t_i^{\zeta_j}$ are such that $|\zeta_{j,e}| \leq w_j|\xi|$ for all time $t \geq 0$. Then, $\xi \equiv 0$ (the origin) is globally asymptotically stable.

Proof: Consider the candidate Lyapunov function $V(\xi) = \xi^T P\xi$ where P satisfies (A1). Utilizing the measurement error interpretation, (8), of the system (6), the derivative of the function V along the flow of the system is

$$\begin{aligned}
\dot{V} &= \xi^T[P\bar{\mathcal{A}} + \bar{\mathcal{A}}^T P]\xi + 2\xi^T P\mathcal{B}\zeta_e^* \\
&\leq -(1-\sigma)\xi^T Q\xi + |\xi|\Big[|2P\mathcal{B}\zeta_e^*| - \sigma Q_m|\xi|\Big] \\
&\leq -(1-\sigma)\xi^T Q\xi + |\xi|\Big[\sum_{j=1}^{q} |2P\mathcal{B}_j\zeta_{j,e}| - \sigma Q_m|\xi|\Big] \\
&\leq -(1-\sigma)\xi^T Q\xi + |\xi|\Big[\sum_{j=1}^{q} |2P\mathcal{B}_j||\zeta_{j,e}| - \sigma Q_m|\xi|\Big]
\end{aligned}$$

The sampling instants have been assumed to be such that the conditions $|\zeta_{j,e}|/|\xi| \leq w_j = \dfrac{\sigma\theta_j Q_m}{|2P\mathcal{B}_j|}$ for each j are satisfied for all time $t \geq 0$. Thus,

$$\dot{V} \leq -(1-\sigma)\xi^T Q\xi$$

which implies that the solution $\xi \equiv 0$ (the origin) is globally asymptotically stable. ■

Note that Lemma 1 holds for a family of asynchronous event-triggers, all satisfying the conditions $|\zeta_{j,e}| \leq w_j|\xi|$. Also note that to enforce these conditions strictly some centralized information, $|\xi|$, is required. Our aim now is to derive realizable decentralized asynchronous event-triggers that belong to the family considered in Lemma 1. Consider the q centralized asynchronous event-triggers for the sampled-data system (6)

$$t_i^{\zeta_j} = \min\Big\{t \geq t_i^{\zeta_j} : |\zeta_{j,e}| \geq w_j|\xi|\Big\}, \quad j \in \{1, \ldots, q\} \tag{10}$$

where w_j are given by (9). From (7) it follows that $|\zeta_j| \leq |\mathcal{K}_j||\xi|$. Thus, enforcing the conditions $|\zeta_{j,e}| \leq w_j|\zeta_j|/|\mathcal{K}_j|$ satisfies the requirements of Lemma 1. Although these conditions utilize only locally available data, they fail to guarantee positive minimum inter-sampling times. In order to design event-triggers that utilize only locally available data while also guaranteeing minimum inter-sample times, let us first analyze the emergent inter-sample times of the centralized asynchronous event-triggers (10).

Let us define the function τ as

$$\tau(w,a,b,k) = \{t \geq 0 : \phi(t,0) = w\} \tag{11}$$

where a, b, k are non-negative constants and $\phi(t,c)$ is the solution of

$$\dot{\phi} = \big(k+\phi\big)\big(a+b\phi\big), \quad \phi(0,c) = c.$$

Note that $\tau(w,a,b,k)$ is positive for any given positive w. The following lemma guarantees positive lower bounds for the emergent inter-sample times for the system (6) with the event-triggers (10).

Lemma 2: Consider the closed loop system defined by (6) along with the event-triggers, (10). Let $w_j > 0$ for $j \in \{1,\ldots,q\}$ be given by (9) and let $W = \sum_{i=j}^{q} |\mathcal{B}_j|w_j$. Then for $j \in \{1,\ldots,q\}$, the inter-sample times $\{t_{i+1}^{\zeta_j} - t_i^{\zeta_j}\}$ are lower bounded by the positive constants

$$T_j = \tau(w_j, |\bar{\mathcal{A}}| + W - |\mathcal{B}_j|w_j, |\mathcal{B}_j|, |\mathcal{K}_j|). \tag{12}$$

Proof: Letting $\nu_j \triangleq |\zeta_{j,e}|/|\xi|$ and by direct calculation we see that for $j \in \{1,\ldots,q\}$

$$\begin{aligned}
\frac{\mathrm{d}\nu_j}{\mathrm{d}t} &= \frac{-(\zeta_{j,e}^T\zeta_{j,e})^{-1/2}\zeta_{j,e}^T\mathcal{K}_j\dot{\xi}}{|\xi|} - \frac{\xi^T\dot{\xi}|\zeta_{j,e}|}{|\xi|^3} \\
&\leq \big(|\mathcal{K}_j| + \nu_j\big)\frac{|\dot{\xi}|}{|\xi|} \\
&\leq \big(|\mathcal{K}_j| + \nu_j\big)\frac{|\bar{\mathcal{A}}\xi| + \sum_{j=1}^{q}|\mathcal{B}_j||\zeta_{j,e}|}{|\xi|}
\end{aligned}$$

where for $\zeta_{j,e} = 0$ the relation holds for all directional derivatives. This relation is further simplified by considering (10), which ensures that the sampling instants are such that for all time $\nu_j \leq w_j$ for each $j \in \{1,\ldots,q\}$.

$$\frac{\mathrm{d}\nu_j}{\mathrm{d}t} \leq \big(|\mathcal{K}_j| + \nu_j\big)\big(|\bar{\mathcal{A}}| + W - |\mathcal{B}_j|w_j + |\mathcal{B}_j|\nu_j\big)$$

from which the claim of the Lemma directly follows. ■

Lemma 2 says that the inter-sample times that emerge from the event-triggers (10) have positive lower bounds, given by (12). An exactly equivalent method of implementing the event-triggers (10) is as follows.

$$t_i^{\zeta_j} = \min\Big\{t \geq t_i^{\zeta_j} + T_j : |\zeta_{j,e}| \geq w_j|\xi|\Big\} \tag{13}$$

In these event-triggers, the lower thresholds for the iner-sample times is explicitly enforced, although the actual inter-sample times that emerge from (13) may have lower bounds greater than T_j. The advantage with this implementation is that T_j depends only on the system matrices and hence is locally known at the corresponding event-trigger. In other words, the j^{th} event-trigger (13) uses only locally available information for time T_j after each of its transmissions. Thus, having guaranteed a positive lower bound for inter-sample times, it is sufficient to under-approximate $|\xi|$ to guarantee global asymptotic stability of the closed loop system. One obvious choice is to use the bound $|\zeta_j|/|\mathcal{K}_j| \leq |\xi|$ in the event-triggers, for $j \in \{1,\ldots,q\}$,

$$t_i^{\zeta_j} = \min\Big\{t \geq t_i^{\zeta_j} + T_j : |\zeta_{j,e}| \geq w_j\frac{|\zeta_j|}{|\mathcal{K}_j|}\Big\}. \tag{14}$$

A better option is to use the bound $|\mathcal{K}_j^+\zeta_j| \leq |\xi|$, where the notation $.^+$ denotes the pseudo-inverse of the matrix. In fact, this is the greatest lower bound for $|\xi|$ given ζ_j. Hence the event-triggers, for $j \in \{1,\ldots,q\}$,

$$t_i^{\zeta_j} = \min\Big\{t \geq t_i^{\zeta_j} + T_j : |\zeta_{j,e}| \geq w_j|\mathcal{K}_j^+\zeta_j|\Big\} \tag{15}$$

use only locally available information and achieve all the design requirements. While the event-triggers we have described in [1], [2] are based on (14), the ones that are described in this paper utilize the improved version (15). Note, however, that if ζ_j is scalar then (14) and (15) are equivalent. The following theorem prescribes the constants T_j and w_j in the event triggers, (15), that guarantee global asymptotic stability of the origin while also explicitly enforcing positive minimum inter-transmission times.

Theorem 1: Consider the closed loop system (8) and assume (A1) holds. Let Q be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q. For each $j \in \{1,2,\ldots,q\}$, let w_j and T_j be defined as in (9) and (12), respectively. Suppose ζ_j are asynchronously transmitted at time instants determined by (15). Then, the origin is globally asymptotically stable and the inter-transmission times are explicitly enforced to have a positive lower threshold.

Proof: The claim about the positive lower threshold for inter-transmission times is obvious from Lemma 2 and (15). Thus, only asymptotic stability remains to be proven. This can be done by showing that the event-triggers (15) are included in the family of event-triggers considered in Lemma 1. From the equivalence of (10) and (13), it is clearly true that $|\zeta_{j,e}| \leq w_j|\xi|$ for $t \in [t_i^{\zeta_j}, t_i^{\zeta_j} + T_j]$, for each $j \in \{1,2,\ldots,q\}$ and each i. Next, for $t \in [t_i^{\zeta_j} + T_j, t_{i+1}^{\zeta_j}]$, (15) enforces $|\zeta_{j,e}| \leq w_j|\mathcal{K}_j^+\zeta_j| \leq w_j|\xi|$. Thus, the event-triggers, (15), are included in the family of event-triggers considered in Lemma 1. Hence, $\xi \equiv 0$ (the origin) is globally asymptotically stable. ■

Next, this general formulation is applied to the dynamic output feedback control over SCAN architecture in Figure 1.

IV. Event-Triggered Dynamic Output Feedback Control Over SCAN

Now, let us consider the design of event-triggered dynamic output feedback control over SCAN architecture of Figure 1. The heart of the SCAN architecture of Figure 1 is the observer layer. Once this is designed, the decentralized asynchronous event-triggers can be designed using the results in Section III. As noted earlier, the nodes in the observer layer do not compute $\hat{x}$ but rather a basis transformation of $\hat{x}$. Defining this transformation is our next task.

(A2) Assume that the column space of the matrix K, in (2), is of dimension m.

Under this assumption, the pseudoinverse $K^+ \in \mathbb{R}^n \times \mathbb{R}^m$ has only the trivial null space. Consider the mapping

$$\hat{x} = K^+ u + \hat{x}^{\mathcal{N}(K)}$$

where $\hat{x}^{\mathcal{N}(K)} \in \mathbb{R}^{n-m}$ is an element of the null space of K and by definition, $K^+ u$ is an element of the row space of K. Assumption (A2) implies that this mapping is one-to-one and onto. Further, since the row space and the null space of K are orthogonal to each other, the basis for the two subspaces can be chosen independently. Thus, let

$$S = \begin{bmatrix} K^+ & K_N \end{bmatrix} \tag{16}$$

where $K_N \in \mathbb{R}^n \times \mathbb{R}^{n-m}$ is an arbitrary matrix whose columns span the null space of K. Then, the matrix S is invertible and satisfies

$$\hat{x} = Sz \tag{17}$$

$$u = u_s^* = KSz_s^* = \bar{K}z_s^*, \text{ with } \bar{K} = \begin{bmatrix} I_m & \mathbf{0}_{m,n-m} \end{bmatrix} \tag{18}$$

where I_m is the $m \times m$ identity matrix and $\mathbf{0}_{m,n-m}$ is $m \times (n-m)$ matrix of zeroes. Even though there is no 'sampling' of the data between the actuator nodes and the plant, the notation u_s^* is useful for keeping in mind that the actuation signals are the asynchronously transmitted signals $\bar{K}z_s^*$. The dynamic controller, (2), is equivalently expressed as

$$\dot{z} = S^{-1}[(A + FC)Sz + B\bar{K}z_s^* - Fy]$$

where $\bar{K} = KS$ has been used.

Letting $H = S^{-1}(A + FC)S$, the sampled data version of the distributed observer is given by

$$\dot{z} = D(H)z + (H - D(H))z_s^* + S^{-1}B\bar{K}z_s^* - S^{-1}Fy_s^*$$

where $D(H)$ is the diagonal matrix with its diagonal entries given by those of the matrix H. It is more convenient to write the observer equation in terms of the sampling induced measurement errors, as follows.

$$\begin{aligned}\dot{z} &= S^{-1}[(A + FC)Sz + B\bar{K}z_s^* - Fy] \\ &\quad + (H - D(H))z_e^* - S^{-1}Fy_e^* \end{aligned} \tag{19}$$

which when expressed in terms of $\hat{x}$ is given as

$$\dot{\hat{x}} = (A + FC)\hat{x} + Bu_s^* - Fy + S(H - D(H))z_e^* - Fy_e^*$$

Let us denote the *observer estimation error* and the state of the closed loop system, respectively, as

$$\tilde{x} \triangleq \hat{x} - x, \quad \psi \triangleq [x^T, \tilde{x}^T]^T$$

Then the closed loop system may be written compactly as

$$\dot{\psi} = \bar{A}\psi + \begin{bmatrix} B\bar{K} \\ S(H - D(H)) \end{bmatrix} z_e^* - \begin{bmatrix} \mathbf{0}_{n,p} \\ F \end{bmatrix} y_e^* \tag{20}$$

where the matrix $\bar{A}$ is as defined in (3). The following theorem prescribes the decentralized asynchronous event-triggering for the control system in Figure 1.

Theorem 2: Consider the closed loop system, (20), and assume that (A1) holds with $\bar{\mathcal{A}} = \bar{A}$. Also suppose (A2) holds. Let $\zeta = [z^T, y^T]^T$ and

$$\mathcal{B} = \begin{bmatrix} B\bar{K} & \mathbf{0}_{n,p} \\ S(H - D(H)) & -F \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} S^{-1} & S^{-1} \\ C & \mathbf{0}_{p,n} \end{bmatrix}.$$

Further, for each $j \in \{1, \dots, q = n + p\}$, let $\zeta_j \in \mathbb{R}$, $\mathcal{B}_j$ is the j^{th} column of $\mathcal{B}$ and $\mathcal{K}_j$ is the j^{th} row of $\mathcal{K}$. Let $Q \in \mathbb{R}^{2n} \times \mathbb{R}^{2n}$ be any symmetric positive definite matrix and let Q_m be the smallest eigenvalue of Q. For each $j \in \{1, 2, \dots, q\}$, let w_j and T_j be defined as in (9) and (12), respectively. Suppose ζ_j are asynchronously transmitted at time instants determined by (15), with $t_0^{\zeta_j} < 0$. Then, $\psi \equiv 0$ (the origin) is globally asymptotically stable and the inter-transmission times are explicitly enforced to have a positive lower threshold.

Proof: Assumption (A2) implies that S is invertible and that the matrices $\mathcal{B}$ and $\mathcal{K}$ are well defined. The rest of the proof follows from Theorem 1. ■

Remark 1: In case the first m nodes of the observer layer, z, are co-located with the corresponding actuator nodes, then $u = \bar{K}z$ may be used. In this case, the closed system equation is given by

$$\dot{\psi} = \bar{A}\psi + \begin{bmatrix} \mathbf{0}_{n,n} \\ S(H - D(H)) \end{bmatrix} z_e^* - \begin{bmatrix} \mathbf{0}_{n,p} \\ F \end{bmatrix} y_e^* \tag{21}$$

and Theorem 2 holds for this system if $\mathcal{B}$ is appropriately chosen as

$$\mathcal{B} = \begin{bmatrix} \mathbf{0}_{n,n} & \mathbf{0}_{n,p} \\ S(H - D(H)) & -F \end{bmatrix}.$$

Remark 2: In Figure 1 and in our results, the sensor nodes and the observer nodes have been assumed to intermittently broadcast their data to all the nodes in the controller/observer layer. However, this has been done purely for ease of presentation. In practice, a sensor node y_j need not transmit its data to the observer node z_k if the dynamics of z_k is not dependent on y_j. A similar statement for intra observer layer communication also holds.

In the next section, simulation results are presented to illustrate the proposed event-triggered controllers.

V. Simulation Results

In this section, the proposed event-triggered dynamic output feedback control over SCAN is illustrated for a linearized model of a batch reactor, [9]. The plant and the dynamic controller are given by (1) and (18)-(19), respectively, with

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$K = -\begin{bmatrix} 0.1768 & 0.079 & 0.0794 & -0.2464 \\ 1.0328 & 0.1896 & -0.4479 & 0.7176 \end{bmatrix}$$

$$F = -\begin{bmatrix} -2 & 0 \\ -4 & -1 \\ -2 & 2 \\ -1 & -4 \end{bmatrix}$$

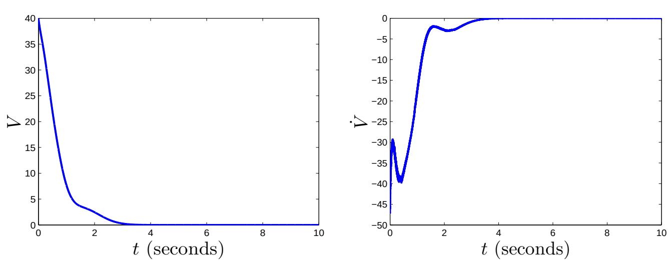
Thus, the closed loop system is given by (20). In the event-triggers, $Q = I_8$, the 8×8 identity matrix, $\sigma = 0.95$ were chosen. For the simulations presented here, the initial condition of the plant and the observer were chosen as $x(0) = [2, 3, -1, 2]^T$ and $z(0) = [0, -1, 1, -1]^T$, respectively. Denoting $\zeta = [z^T, y^T]^T$ as in Theorem 2, the initial sampled data was chosen arbitrarily as

$$\zeta_s^*(0) = [-1.001, -1.001, 1.001, -1.001, -1.001, 3.002]^T$$

so that it is consistent with the asynchronous transmission model. The zeroth transmission instant was chosen as $t_0^{\zeta_j} = -T_j$ for each $j \in \{1, \ldots, 6\}$. This is to ensure sampling at $t = 0$ if necessary. However, by choosing the initial sampled data sufficiently close to the actual data, the asynchronous nature of transmissions is respected, as indicated by the first transmission times by the 6 nodes which occur at $t_1^{\zeta} = [6, 1.1, 0.4, 1.2, 0.4, 0.9]$ms for the chosen initial conditions. The inter-transmission time thresholds in the event-triggers, (15), were obtained as

$$T = 10^{-4} \times [4.886, 4.676, 5.247, 3.976, 4.12, 3.881]\text{s}$$

which were also the minimum inter-transmission times for the presented simulation. Over a simulation time of 10s, the average inter-transmission times for the nodes were obtained as $\bar{T} = [3.1, 3, 2.7, 2.6, 2.7, 3]$ms, which are roughly an order of magnitude larger than the inter-transmission time thresholds. Figure 2 shows the evolution of the Lyapunov function and its derivative along the flow of the closed loop system. Figure 3 shows the inter-transmission times and the cumulative frequency distribution of the inter-transmission times of the nodes.

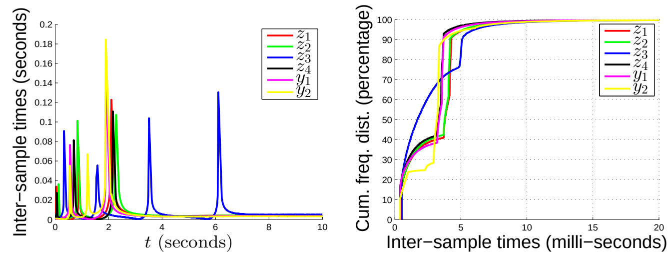


(a) (b)

Fig. 2: (a) The evolution of the Lyapunov function and (b) its derivative along the flow of the closed loop system.

VI. Conclusions

This paper presents the design of event-triggered output feedback control of LTI systems over Sensor-Controller-Actuator Networks (SCAN). A SCAN is divided into three functional layers - sensor layer, controller/observer layer and the actuator layer, each layer consisting of several nodes. The communication between the nodes is intermittent and event-triggered. Further, the flow of information is only from the sensor to observer to actuator layer with the only intra-layer communication occurring in the observer layer. With a careful choice of basis for distributed estimation of the plant state in the observer layer, each actuator node intermittently receives data from a corresponding unique observer node. The event-triggers are designed to utilize only locally available information, making the nodes' transmissions asynchronous. The proposed design guarantees global asymptotic stability of the origin of the system and a positive lower bound for the inter-transmission times of each node individually. The proposed design methodology was illustrated through simulations of a linearized model of a batch reactor. Some of the future work will include relaxation of assumption (A2), extending the design to the case where an arbitrary communication graph is given and optimal placement of the controller/observer nodes (see Remark 1 for example).



(a) (b)

Fig. 3: (a) Inter-transmission times and (b) the cumulative frequency distribution of the inter-transmission times of the nodes. The curves labelled with z_i and y_j denote the relevant inter-transmission time data of those nodes, respectively.

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